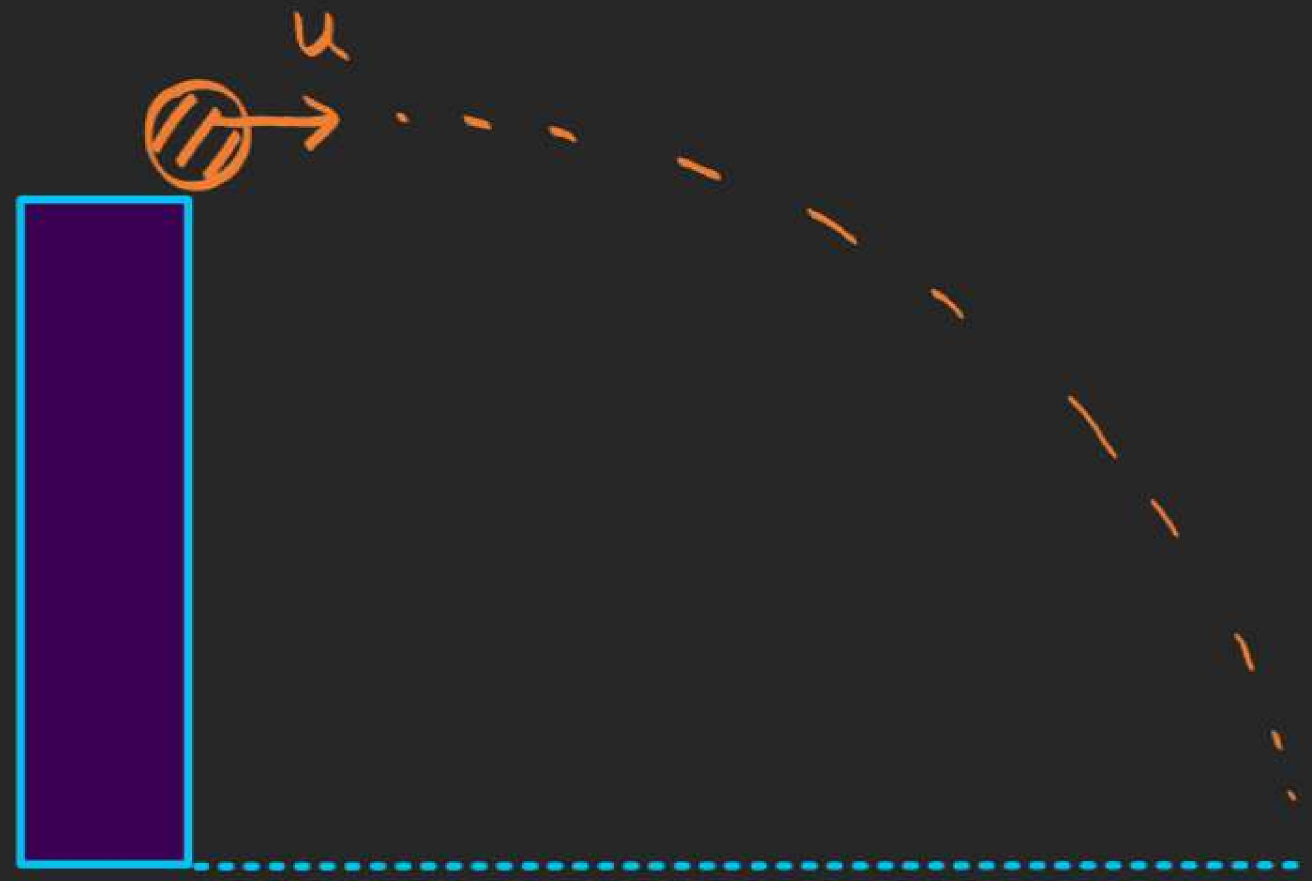
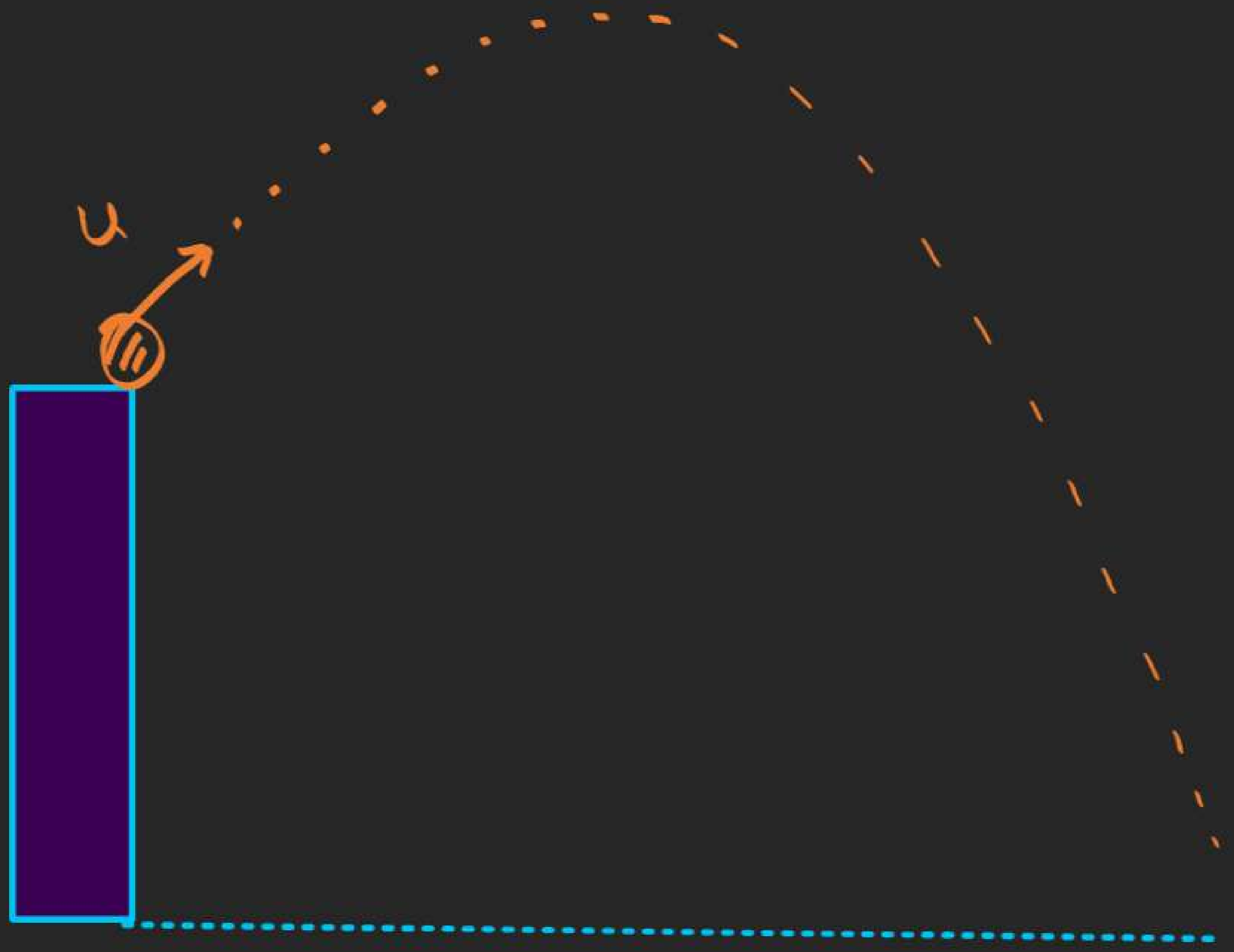
Projectile Motion

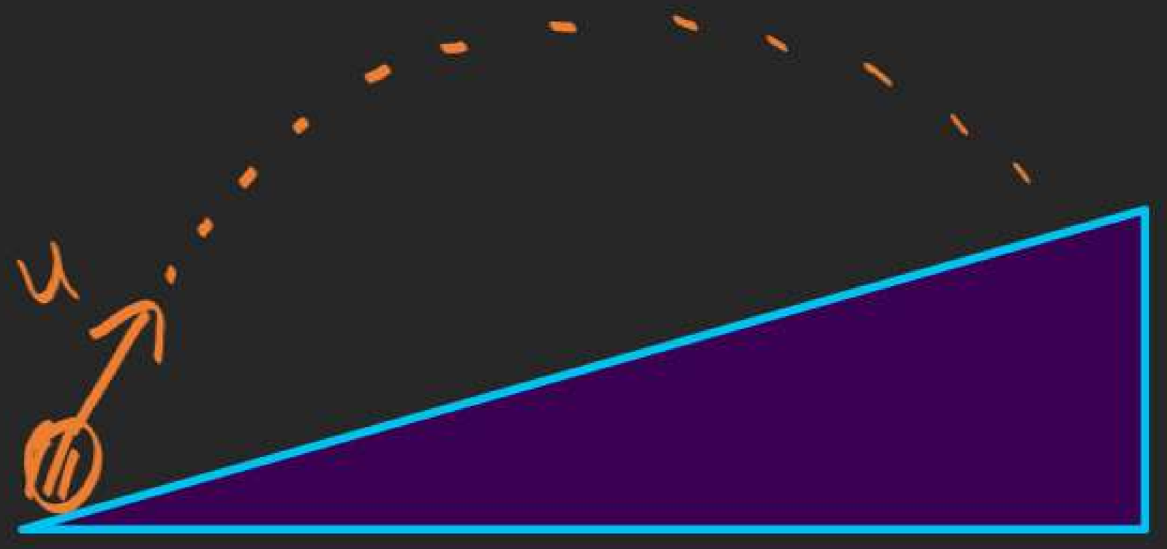
a) Ground to Ground

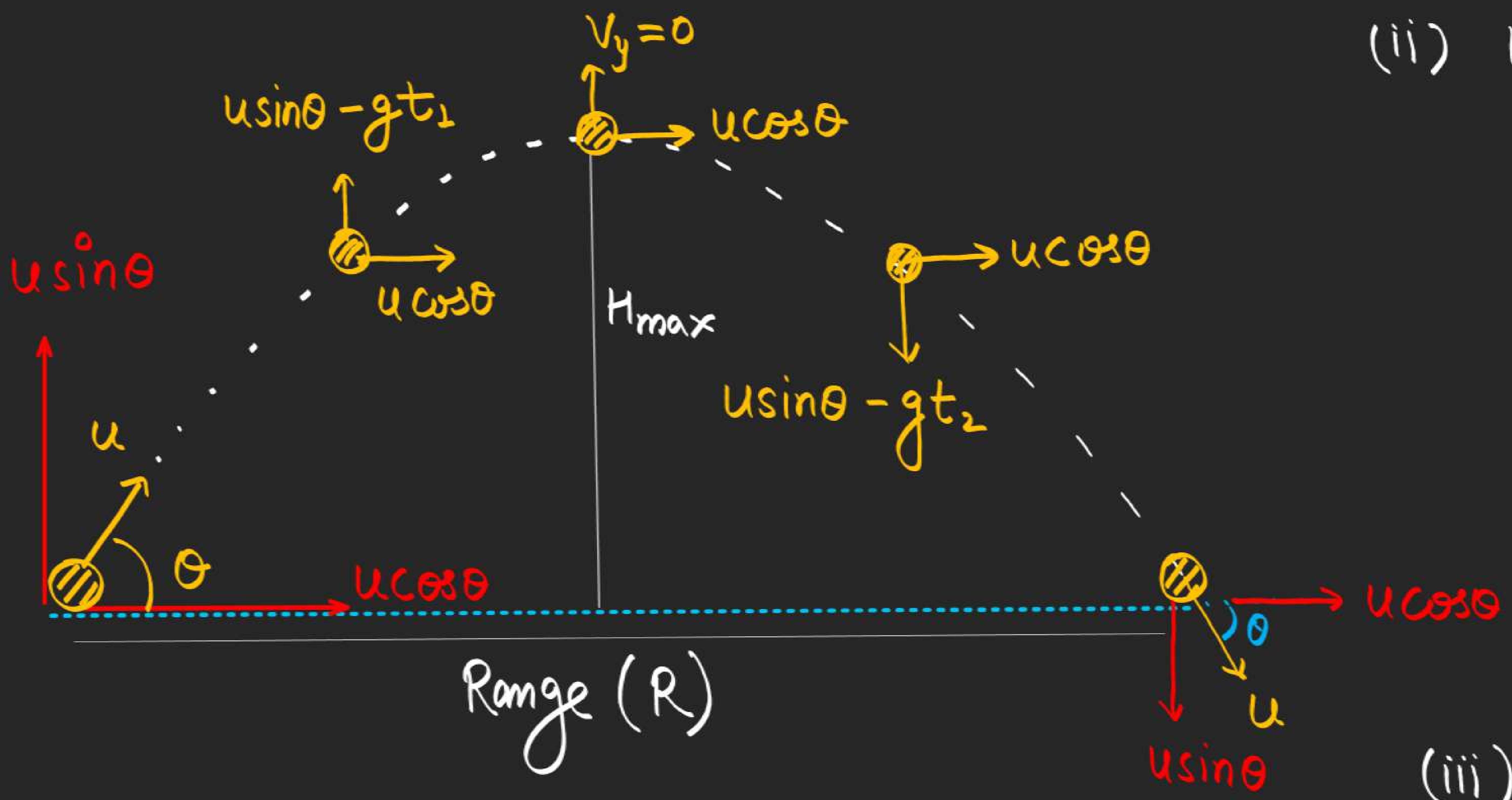
b) From height (Horizontal projectile)





c) Inclined plane





(i) <u>At any time</u>

Angle with horizontal

$$\tan\theta = \left(\frac{V_y}{V_x}\right) = \left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

(ii) H_{max} : using eq^n motion under gravity

$$v^2 = u^2 - 2gH$$

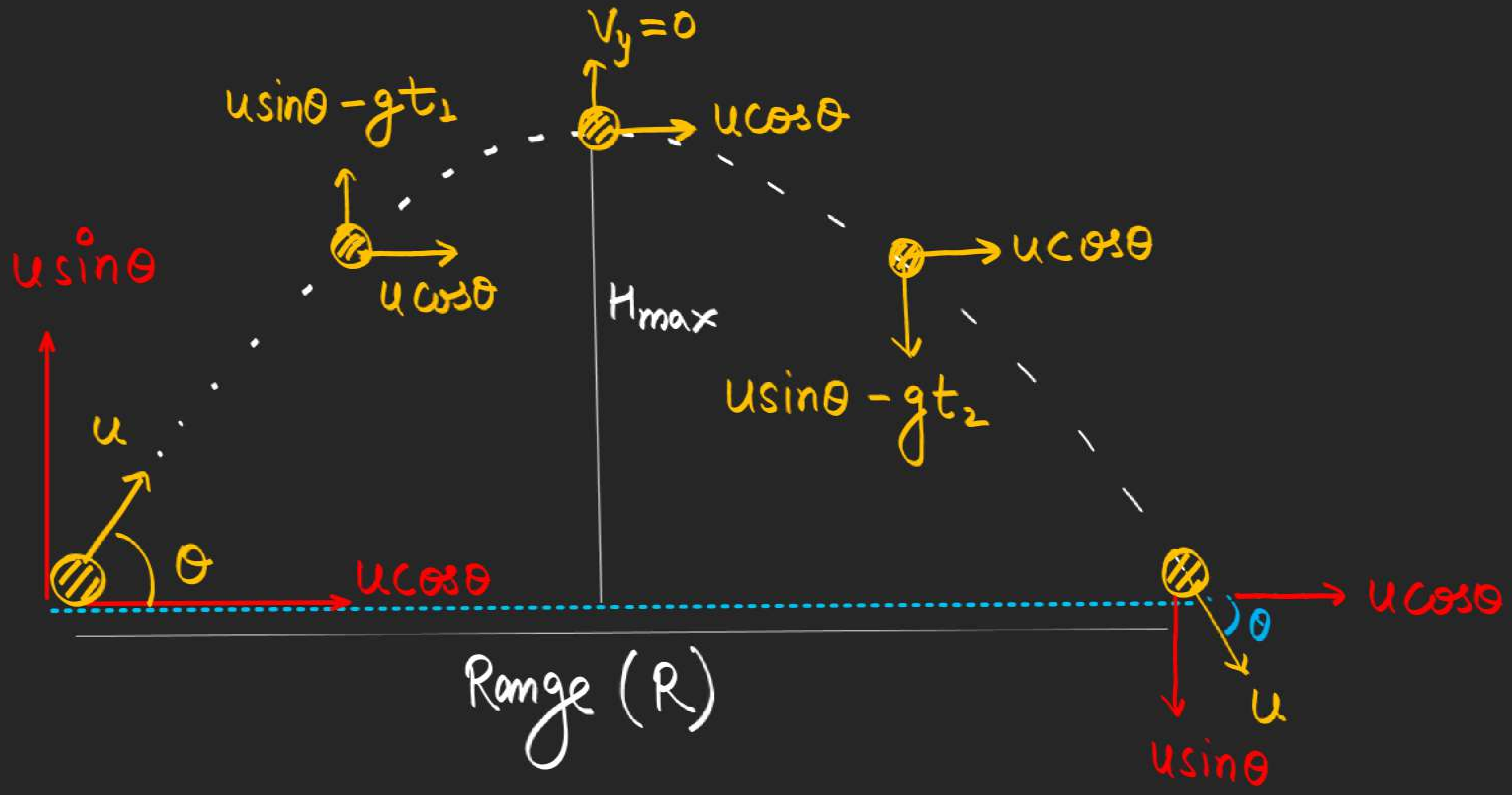
$$0 = (u\sin\theta)^2 - 2gH$$

$$\boxed{H = \frac{(u\sin\theta)^2}{2g}}$$

(iii) <u>Time of flight</u>

using motion under gravity

$$\boxed{t = \frac{2U_y}{g} = \left(\frac{2u\sin\theta}{g}\right)}$$



(iv) Range : max horizontal distance

$$R = u_x \cdot T_f$$

$$= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Imp: Range is max when $\sin 2\theta = 1$

$$2\theta = \pi/2$$

$$\boxed{\theta = \pi/4}$$

Imp: Range is same at (θ) and $(90-\theta)$

Also at $(45+\theta)$ & $(45-\theta)$

(V) Relation b/w R and H

$$R = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

divide

$$\frac{R}{H} = 4\cot\theta$$

$$\Downarrow$$

$$\boxed{R_{max} = 4H}$$

when $\theta = 45^\circ$

$$R = \frac{1}{2} g t_1 t_2$$

Proof

$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2}$$

$$t_1 t_2 = \frac{2}{g}\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)$$

(θ) $\quad \frac{R}{H_1} = 4\cot\theta$

$(90-\theta)$ $\quad \frac{R}{H_2} = 4\cot(90-\theta)$

multiply $\quad \frac{R^2}{H_1 H_2} = 16$

$$R^2 = 16 H_1 H_2$$

Divide $\quad \frac{H_2}{H_1} = \cot^2\theta$

Example At $\pi/6$ angle of projection Range is 50m.

i) **Max Height** : $\frac{R}{H} = 4\cot\theta \Rightarrow \frac{50}{H} = 4\cot\frac{\pi}{6} \Rightarrow H = \frac{25}{2\sqrt{3}}$ m

ii) **Velocity of projectile (u)** : $R = \frac{u^2 \sin 2\theta}{g}$ $\Rightarrow 50 = \frac{u^2 \sin 60^\circ}{10}$

$\Rightarrow \frac{1000}{\sqrt{3}} = u^2$

(iii) If with same velocity thrown at 60° find range, Height

Ans Range $\rightarrow$ same $= 50$m

$\Rightarrow R^2 = 16 H_1 H_2$ or $\frac{H_2}{H_1} = \cot^2\theta \Rightarrow H_2 = H_1 \cot^2\theta$

$\Rightarrow 2500 = 16\left(\frac{25}{2\sqrt{3}}\right) H_2$; $= \frac{25}{2\sqrt{3}} \times (\sqrt{3})^2 = \left(\frac{25\sqrt{3}}{2}\right)$

$\Rightarrow \frac{25\sqrt{3}}{2} = H_2$

Q If Deepak can through a ball to 100m horizontally. Find H_{max} he can through

Ans $R_{max} = \frac{u^2 \sin 2\theta}{g} \Rightarrow 100 = \frac{u^2}{g}(1)$ at $\theta = \pi/4$

$$\boxed{100 = \frac{u^2}{g}}$$

Now H_{max} he can thrown when thrown fully vertical

$$H_{max} = \frac{u^2}{2g} = \frac{1}{2} \times 100$$

$$= \underline{\underline{50m}}$$

Ex $u = 40\ m/s$ at 30° angle of Projection

find H_{max}, T_f, R, Velocity at $t = 3/2\ s$ and at $3\ s$, angle with horizontal at $t = 3\ s$.

Also find coordinate at $t = 2\ sec$. If thrown From origin

Also find avg velocity in journey

Ans: $H_{max} = \dfrac{(u\sin\theta)^2}{2g}$

$= \dfrac{\left(40\times\frac{1}{2}\right)^2}{20}$

$= \underline{\underline{20m}}$

$R = \dfrac{u^2\sin2\theta}{g}$

$= \dfrac{1600}{10}\times\dfrac{\sqrt{3}}{2}$

$= 80\sqrt{3}\ m$

$T_f = \dfrac{2u\sin\theta}{g}$

$= \dfrac{2\times40}{10}\times\dfrac{1}{2}$

$= 4\ sec$

Ex $u = 40\,m/s$ at 30° angle of Projection

find H_{max}, T_f, R, velocity at $t = 3/2$ s and at $3s$, angle with horizontal at $t = 3s$.

Also find coordinate at $t = 2$ sec. If thrown from origin

Also find avg velocity in journey

(iv)

$u\sin\theta = 40 \times \frac{1}{2}$

$= 20\,m/s$

$u\cos\theta$

$V_x = 40 \times \frac{\sqrt{3}}{2}$

$V_x = 20\sqrt{3}\,m/s$ ----- Const.

at 3/2 sec

Vertical $\Rightarrow V_y = u\sin\theta - gt$

$= 20 - 10 \times \frac{3}{2}$

$= 5\,m/s\ \hat{j}$

Horizontal $\Rightarrow V_x = 20\sqrt{3}\ \hat{i}$

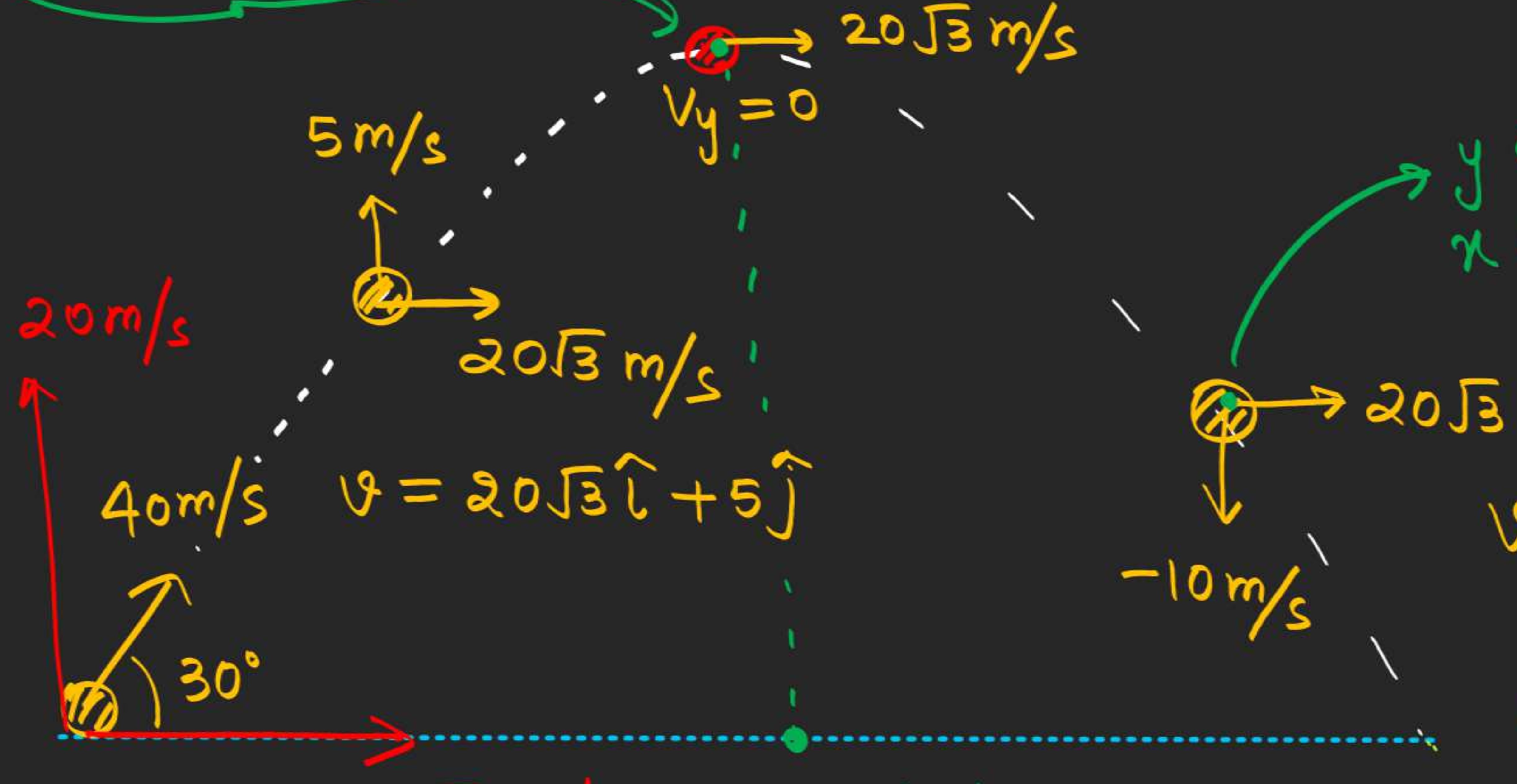
$\vec{V}_{net} = 20\sqrt{3}\,\hat{i} + 5\hat{j}$

$V = \sqrt{1200 + 25}$

$= \underline{\underline{35\,m/s}}$

$y = H_{max} = 20$

$t = 2\,sec$



$20\sqrt{3}\ m/s$

y cordinate $= 15$

x cordinate $= V_x t = 20\sqrt{3} \times 3 = 60\sqrt{3}$

$v = 20\sqrt{3}\,\hat{i} - 10\,\hat{j}$

Angle with horizontal

$$\tan\theta = \frac{V_y}{V_x} = \frac{10}{20\sqrt{3}}$$

$$\tan\theta = \frac{1}{2\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

$x = V_x \cdot t$

$x = 20\sqrt{3} \times 2$

$= 40\sqrt{3}$

$V_{avg} = \frac{disp.}{time} = \frac{80\sqrt{3}}{4} = 20\sqrt{3}\ m/s$

Eqn of Trajectory

At any time (t)

$$x = (u\cos\theta)t \quad \cdots\to \quad t = \frac{x}{u\cos\theta} \qquad \text{———} \; \mathrm{I}$$

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$y = (\cancel{u}\sin\theta)\left(\frac{x}{\cancel{u}\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$= x\tan\theta - \frac{x^2 \sin\theta}{\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)\cos\theta}$$

$$= x\tan\theta - \frac{x^2\tan\theta}{R} \implies \boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Time when $\vec{u_i}$ and $\vec{v_f}$ are $\perp$r

$$\vec{u_i} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

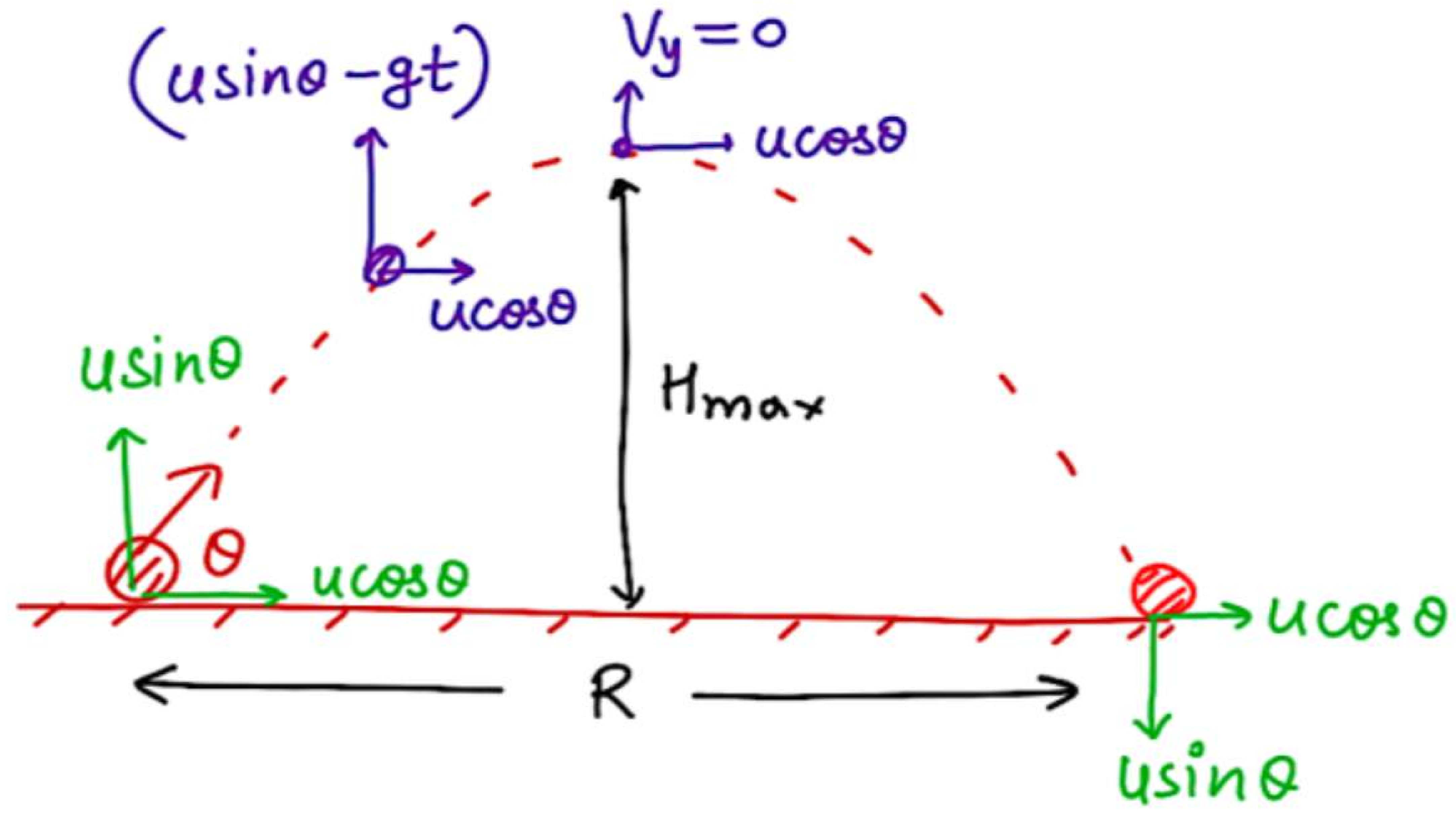
$$\vec{v_f} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

$$\vec{u_i}\cdot\vec{v_f} = u^2\cos^2\theta + u^2\sin^2\theta - ugt\sin\theta$$

$$0 = u^2(\cos^2\theta + \sin^2\theta) - (ug\sin\theta)t$$

$$(\cancel{u}g\sin\theta)t = u^{\cancel{2}}$$

$$\boxed{t = \frac{u}{g\sin\theta}}$$



i) $T = \frac{2u\sin\theta}{g}$ ii) $H = \frac{(u\sin\theta)^2}{2g}$

iii) $R = V_x \cdot T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$

iv) $R_{max}\ (45°)$ (v) $R_1 = R_2$ when $\theta,\ 90-\theta$
or $\frac{\pi}{4}-\theta,\ \frac{\pi}{4}+\theta$

vi) $\frac{R}{H} = 4\cot\theta \implies R = 4H_1\cot\theta$
$R = 4H_2\cot(90-\theta)$

$\boxed{R^2 = 16H_1H_2}$

vii) $R_{max} = 4H$

(*) viii) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$\boxed{\tan^2\theta = \frac{H_1}{H_2}}$

$\boxed{R = \frac{1}{2}gT_1T_2}$

(ix) V_i & V_f are ⊥r
$\left(t = \frac{u}{g\sin\theta}\right)$

(*) Rain Umbrella Problem
↳ vector digm solve

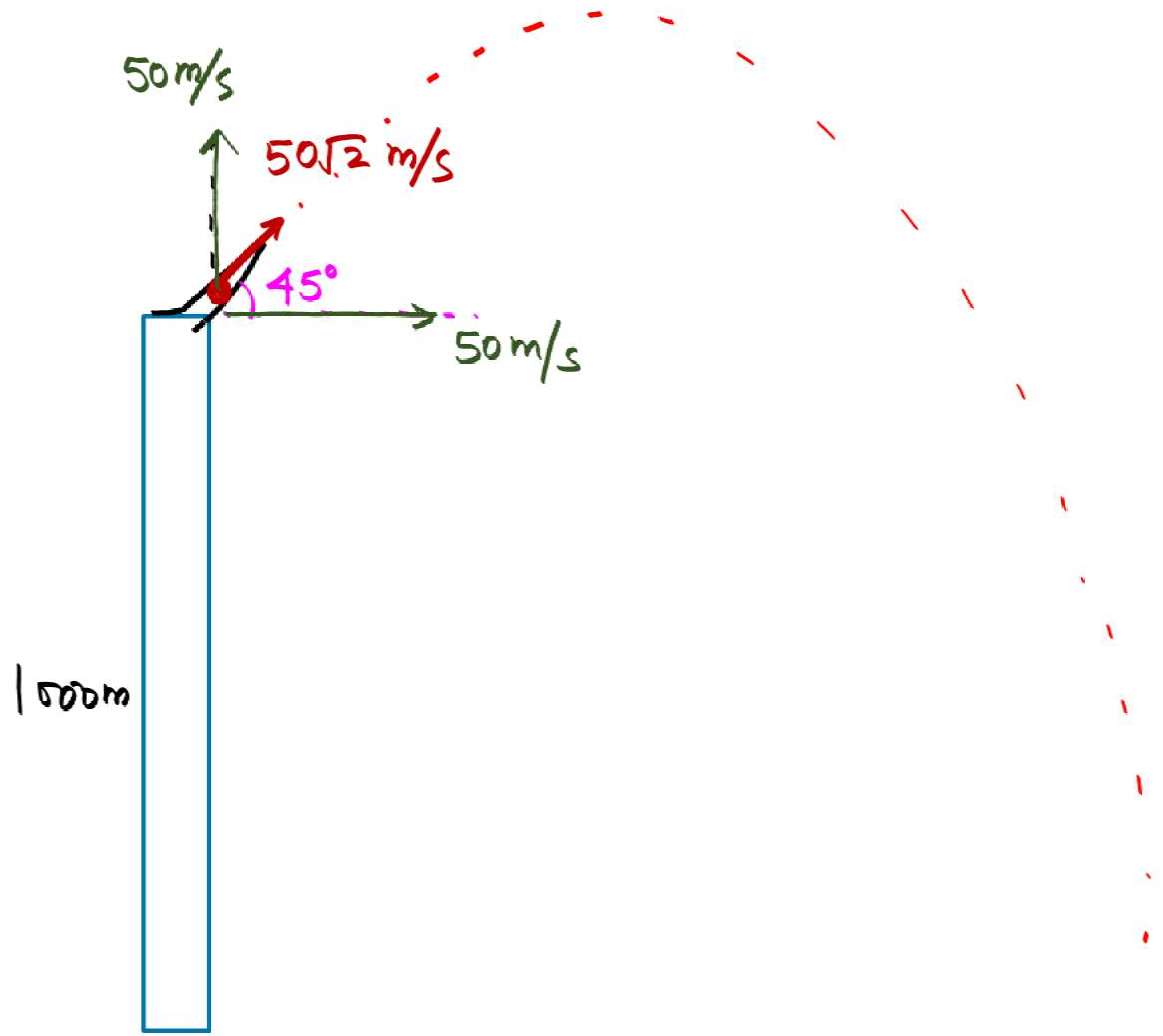
(*) River swimmer Problem
↳ min time $(t) = \frac{d}{V_m}$, Drift $= V_R \cdot t_{min}$
↳ min distance :

A fighter plane moving with a speed of $50\sqrt{2}$ m/s upward at an angle of 45° with the vertical, releases a bomb from height 1000 m above the ground. Find

(a) time of flight

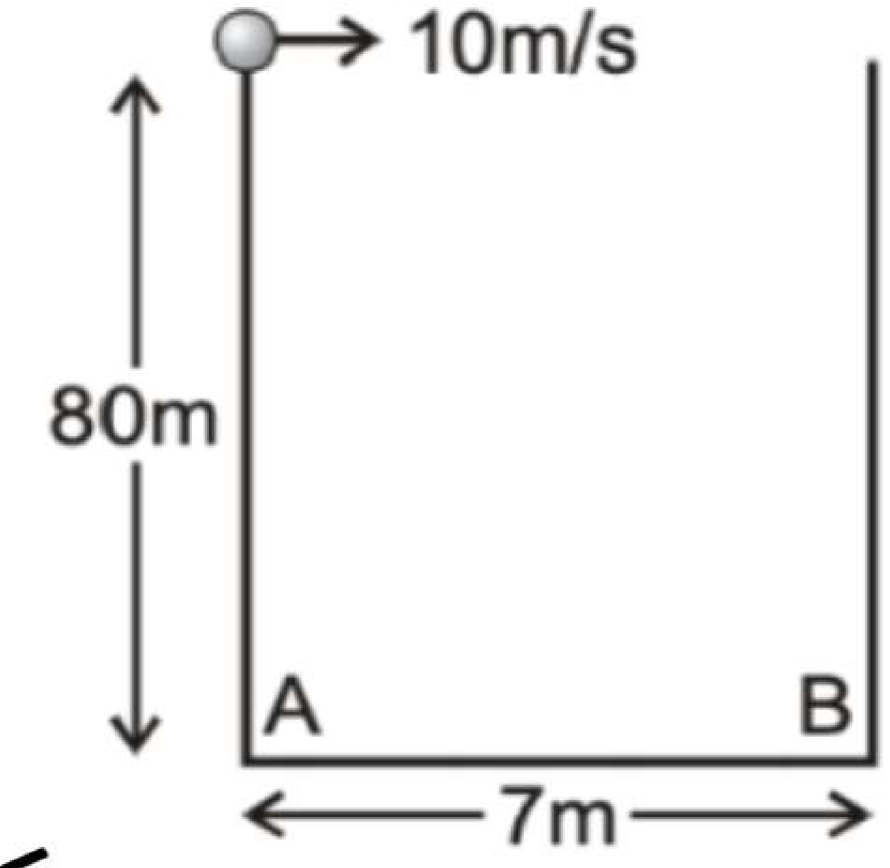
(b) maximum height of the bomb above ground

HW



A ball is projected horizontally from top of a 80 m deep well with velocity 10 m/s. Then particle will fall on the bottom at a distance of (all the collisions with the wall are elastic).

Adv



(1) 5 m from A ~~(2) 5 m from B~~ ~~(3) 2 m from A~~ (4) 2 m from B

(i) $H = \frac{1}{2}gt^2$

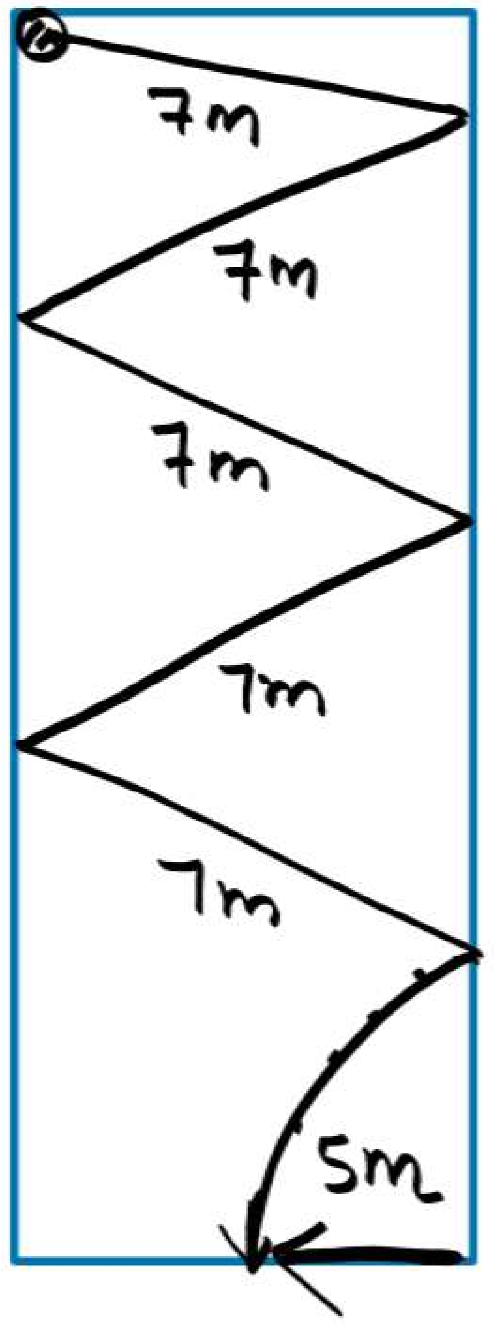
$80 = 5t^2$

$\boxed{4 = t}$

(ii) Range $= V_x T$

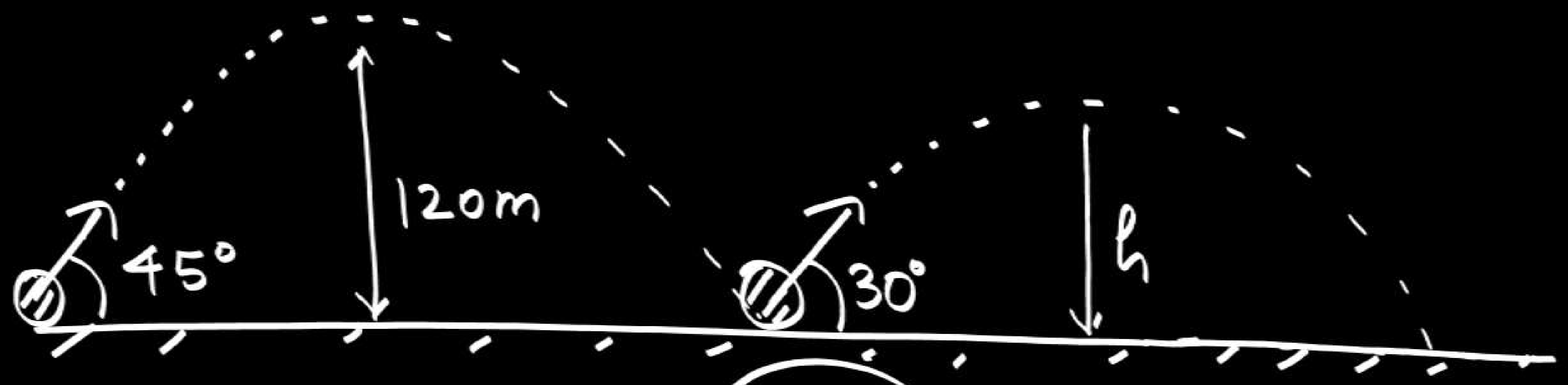
$= 10 \times 4$

$= \underline{40m}$



A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce in metres, is ________.

[JEE (Advanced) 2018]



$$H = \frac{(u\sin\theta)^2}{2g}$$

$$120 = \frac{u^2}{4g}$$

$$4800 = u^2$$

KE is Halved

$\frac{1}{2}mv^2 \rightarrow$ Half

$u \rightarrow \frac{1}{\sqrt{2}}$ times

$u^2 \rightarrow \frac{1}{2}$ times $= 2400$

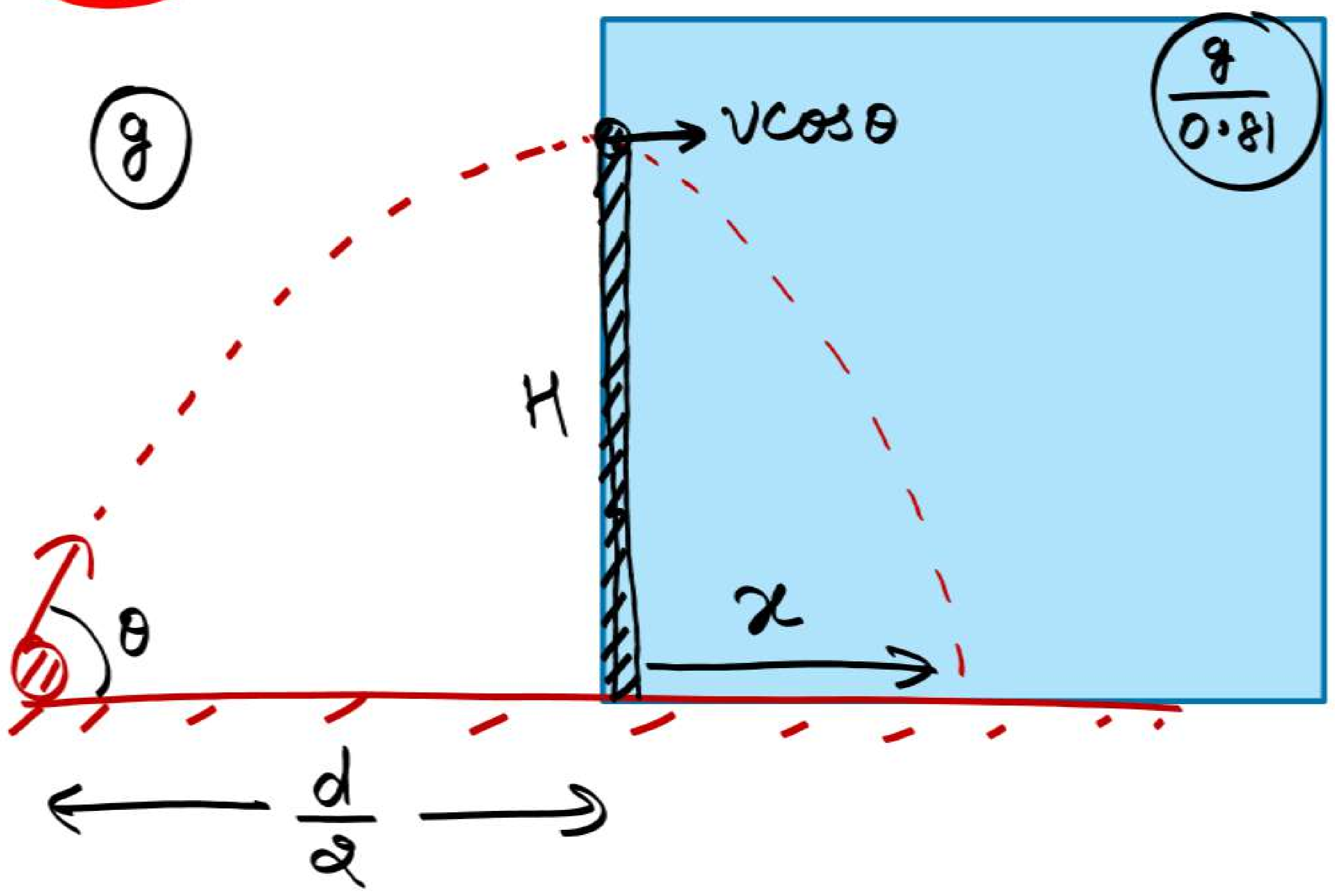
$$\Rightarrow h = \frac{(u\sin\theta)^2}{2g} = \frac{u^2 \times \frac{1}{4}}{20} = \frac{2400 \times \frac{1}{4}}{20} = 30\text{m}$$

Correct answer is 30

A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is $d' = nd$. The value of n is ___________ .

$$\Rightarrow \sqrt{gg'} = \sqrt{g \times \frac{g}{0.81}} = \frac{g}{0.9}$$

$$d' = 0.95\, d$$



$$x = (v\cos\theta)\, t = (v\cos\theta)\left(\frac{v\sin\theta}{g}\right) \times 0.9$$

$$= \frac{\frac{2v^2 \sin\theta\cos\theta}{g}}{2} \times 0.9$$

$$= \frac{d}{2}(0.9) = 0.45d$$

$$H = \frac{(v\sin\theta)^2}{\cancel{2}g} = \frac{1}{\cancel{2}} g' t^2 \Rightarrow \frac{v\sin\theta}{\sqrt{gg'}} = t \Rightarrow \frac{v\sin\theta}{g} \times 0.9 = t$$

, $n = 0.95d$

The equation of projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$. The speed and angle of projection are

$$y = \sqrt{3}x\left(1 - \frac{x}{\left(\frac{2\sqrt{3}}{g}\right)}\right) \xrightarrow{\text{Compare}} y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$R = \frac{2\sqrt{3}}{g}$$

$$\tan\theta = \sqrt{3}$$

$$\boxed{\theta = \pi/3}$$

$$\# \quad R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$\frac{\cancel{2\sqrt{3}}}{\cancel{g}} = \frac{\cancel{2}u^2}{\cancel{g}}\left(\frac{\cancel{\sqrt{3}}}{\cancel{2}}\right)\left(\frac{1}{2}\right)$$

$$4 = u^2$$

$$\boxed{2 = u}$$

$2, 60^o$

JEE Main 2019 (Online) 12th April Morning Slot

MCQ Single Correct Answer

The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10$ ms^{-2}) :

(A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

(B) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

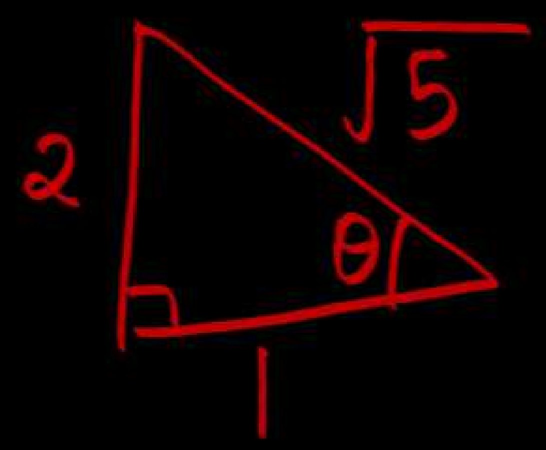
(C) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

(D) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

$$y = 2x\left(1 - \frac{9x}{2}\right)$$

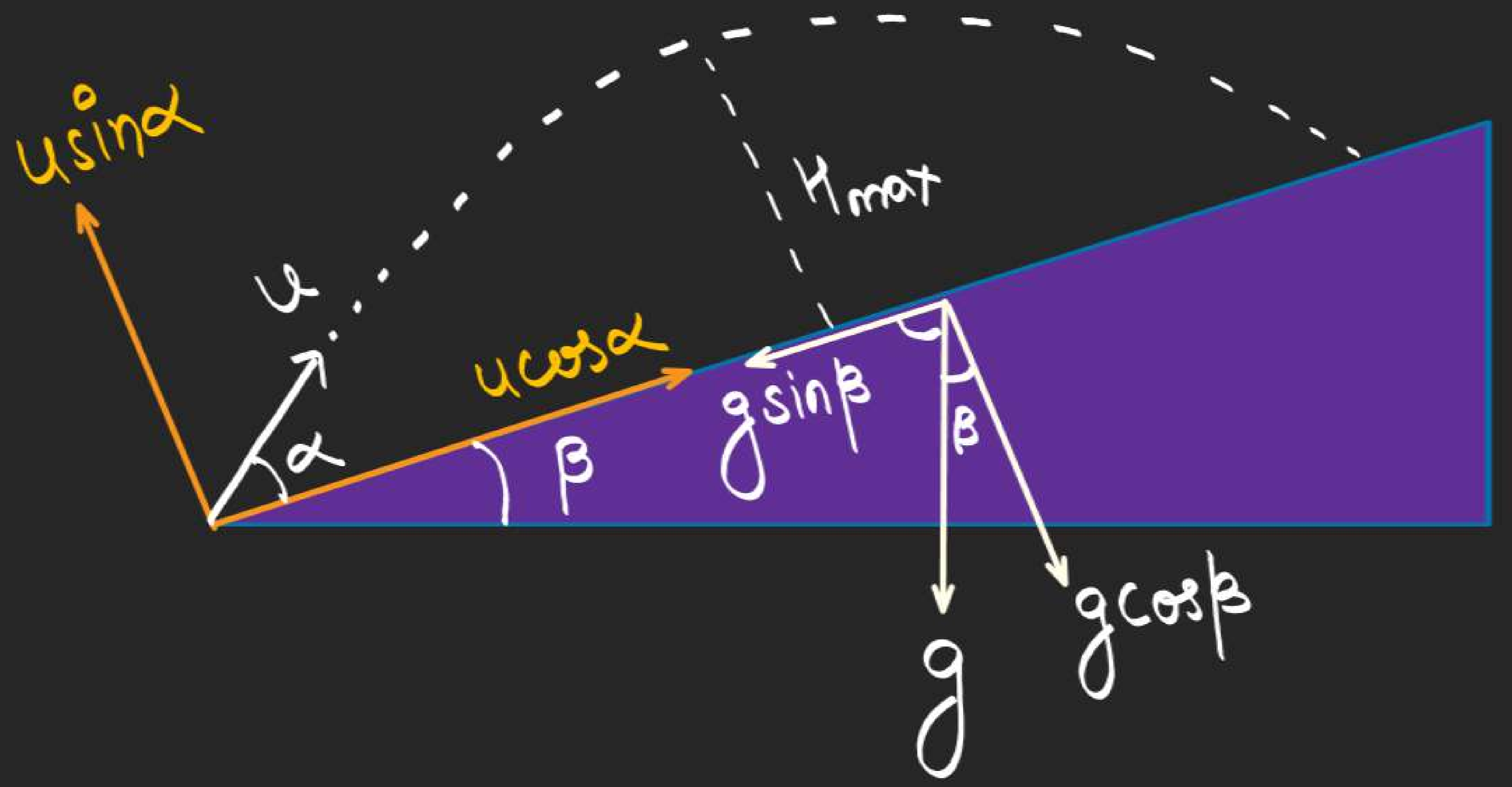
$$= 2x\left(1 - \frac{x}{2/9}\right)$$

$\tan\theta = 2$, $R = 2/9$



$$\frac{\cancel{2}v^2 \sin\theta\cos\theta}{g} = \frac{\cancel{2}}{9}$$

$$\frac{v^2}{10} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{9}$$



(i) $$H_{max} = \frac{(u\sin\alpha)^2}{2g\cos\beta}$$

(ii) $$T = \frac{2u\sin\alpha}{g\cos\beta}$$

iii) $$R = (u\cos\alpha)T - \frac{1}{2}g\sin\beta\,T^2$$

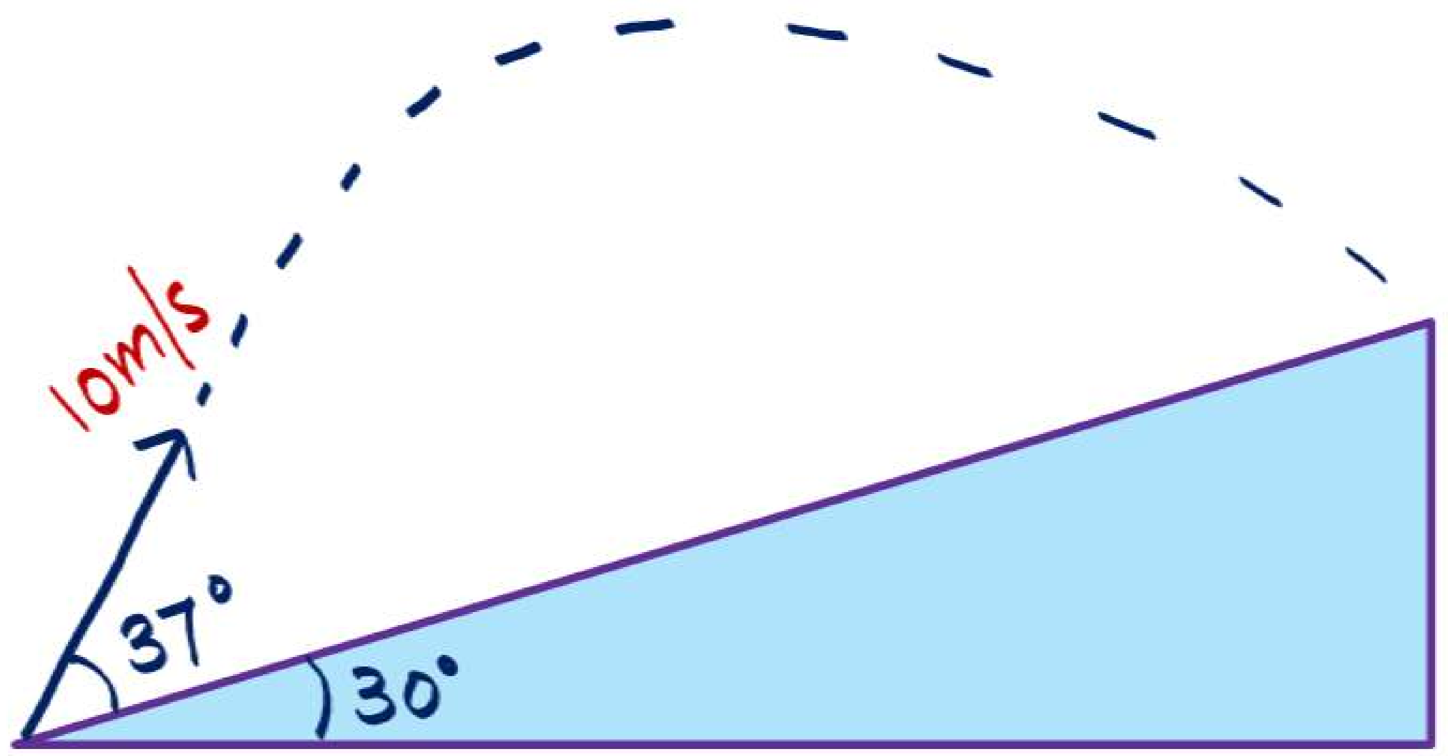
$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$

Standard results for projectile motion on an inclined plane

	Up the Incline	Down the Incline
Range	$\dfrac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$	$\dfrac{2u^2 \sin\alpha \cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\dfrac{2u\sin\alpha}{g\cos\beta}$	$\dfrac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\alpha = \dfrac{\pi}{4}-\dfrac{\beta}{2} = \dfrac{1}{2}\left(\dfrac{\pi}{2}-\beta\right)$	$\alpha = \dfrac{\pi}{4}+\dfrac{\beta}{2} = \dfrac{1}{2}\left(\dfrac{\pi}{2}+\beta\right)$
Maximum Range	$\dfrac{u^2}{g(1+\sin\beta)}$	$\dfrac{u^2}{g(1-\sin\beta)}$

Here α is the angle of projection with the incline and β is the angle of incline.

A bullet is fired from the bottom of the inclined plane at angle θ = 37° with the inclined plane. The angle of incline is 30° with the horizontal. Find (i) the position of the maximum height of the bullet from the inclined plane. (ii) Time of light (iii) Horizontal range along the incline. (iv) For what value of θ will range be maximum. (v) Maximum range.



(i) $H_{max} = \dfrac{(u\sin\alpha)^2}{2g\cos\beta} = \dfrac{\left(10\times\frac{3}{5}\right)^2}{2\times10\times\frac{\sqrt{3}}{2}} = \dfrac{36}{10\sqrt{3}}$ m

(ii) $T_f = \dfrac{2u\sin\alpha}{g\cos\beta} = \dfrac{2\times6}{10\times\frac{\sqrt{3}}{2}} = \dfrac{24}{10\sqrt{3}}$ sec

(iii) $R = \dfrac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta} = \dfrac{2\times100\times\frac{3}{5}\times\cos 67}{10\times\frac{3}{4}}$

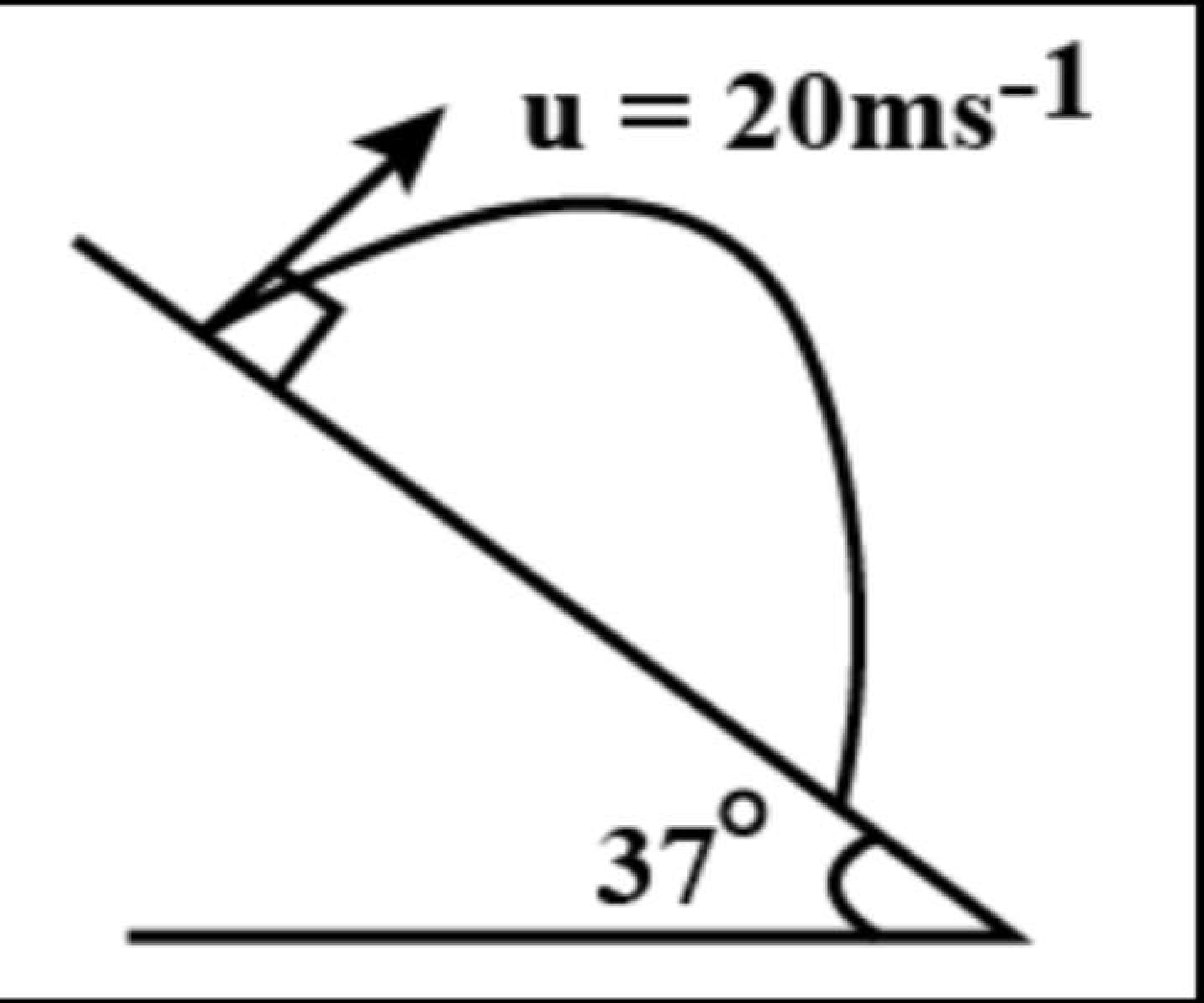
(IV) $\alpha = \frac{1}{2}\left(\pi/2-\beta\right)$

$= \frac{1}{2}\times(90-30)$

$= 30°$

(V) $\dfrac{u^2}{g(1+\sin\beta)} = \dfrac{100}{10\times\frac{3}{2}} = 20/3$ m

Find range of projectile on the inclined plane which is projected perpendicular to the inclined plane with velocity 20 m/s as shown in figure :



$$R = \frac{2u^2 \sin\alpha \cos(\alpha-\beta)}{g\cos^2\beta}$$

$$= \frac{2(4\cancel{00})\sin 90^\circ \cos(90-37^\circ)}{1\cancel{0} \times \left(\frac{4}{5}\right)^2}$$

$$= \frac{\cancel{80} \times \frac{3}{\cancel{5}}}{\cancel{16}/25} = 75\,m$$

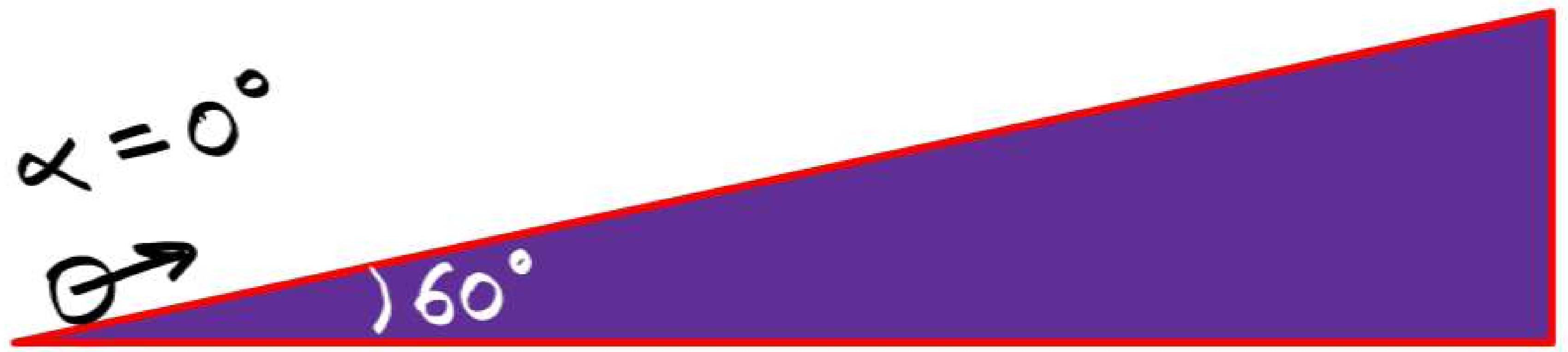
LIVE

When an object is shot from the bottom of a long smooth inclined plane kept at an angle $60°$ with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to $30°$ and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be

(a) $1 : 2\sqrt{3}$ (b) $1 : \sqrt{2}$

(c) $\sqrt{2} : 1$ (d) $1 : \sqrt{3}$

(NEET 2019)

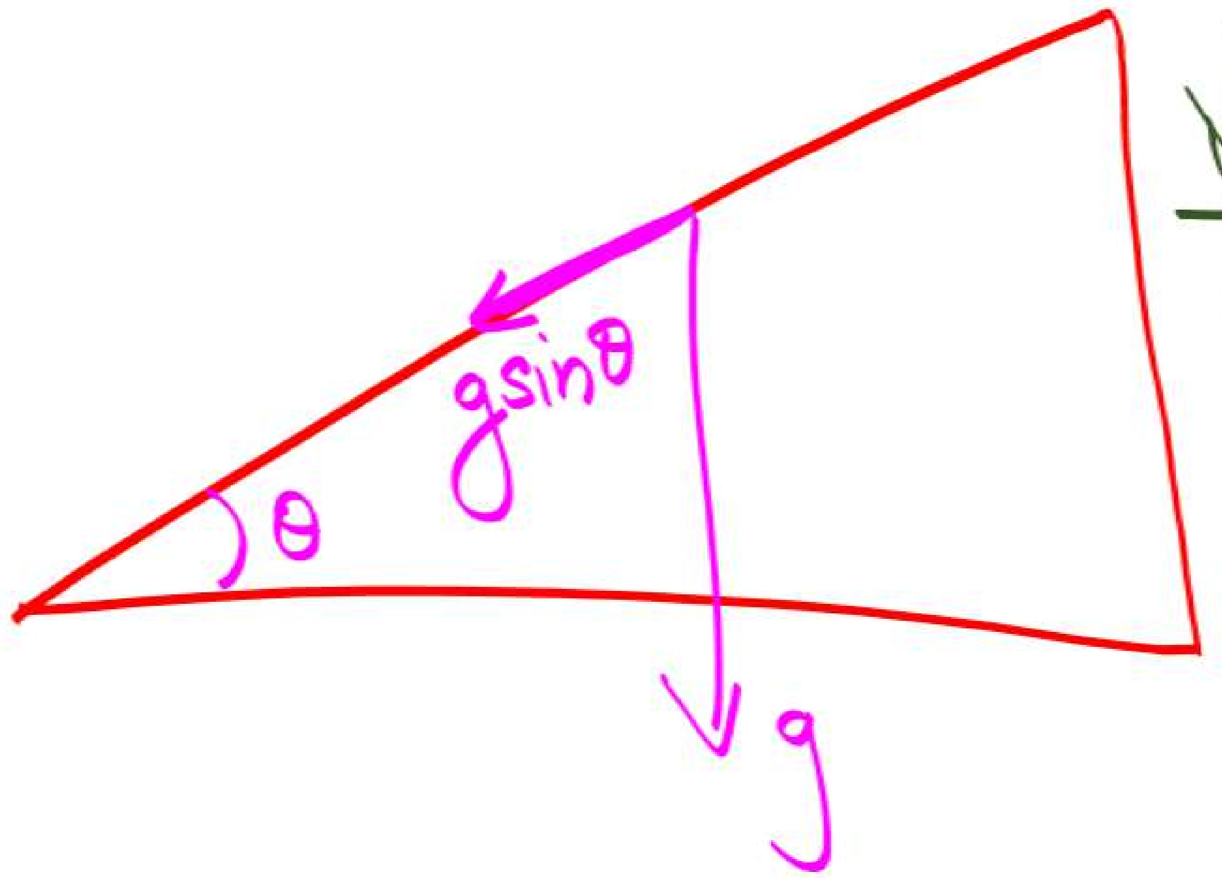


$$x_1 = \frac{\cancel{2v^2 \sin\alpha} \cos(\alpha + 60°)}{\cancel{g} \cos^2 60°} = \frac{1/2}{1/4}$$

$$x_2 = \frac{\cancel{2v^2 \sin\alpha} \cos(\alpha + 30°)}{\cancel{g} \cos^2 30°} \qquad \frac{\sqrt{3}/2}{3/4}$$

$$= \frac{2}{2/\sqrt{3}}$$

$$= \frac{\sqrt{3}}{1}$$



$$\frac{\cancel{v_1}}{\cancel{v_2}} = \frac{\sqrt{\cancel{2g} \sin 60° \, x_1}}{\sqrt{\cancel{2g} \sin 30° \, x_2}}$$

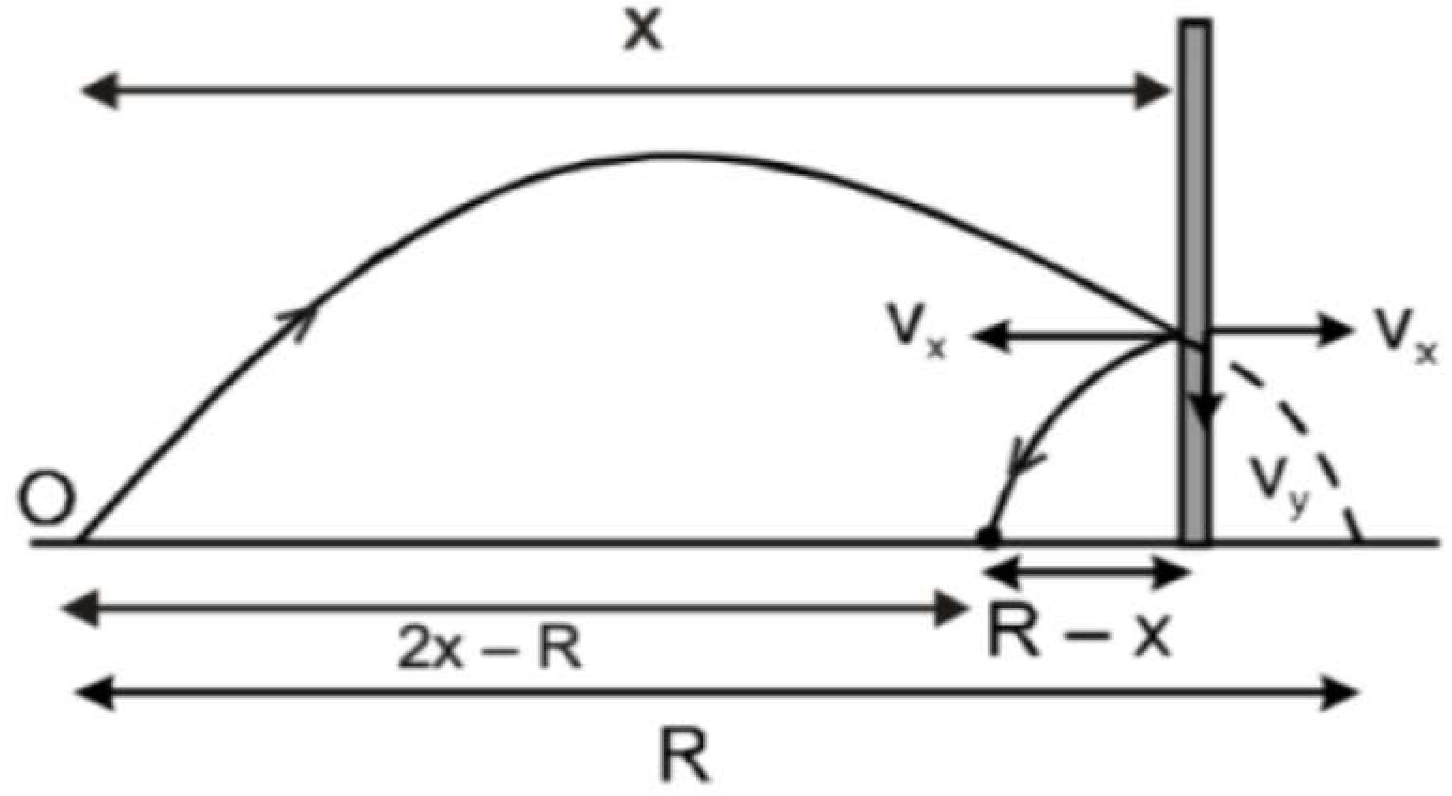
$$1 = \frac{\sqrt{3}}{2 \times \frac{1}{2}} \frac{x_1}{x_2}$$

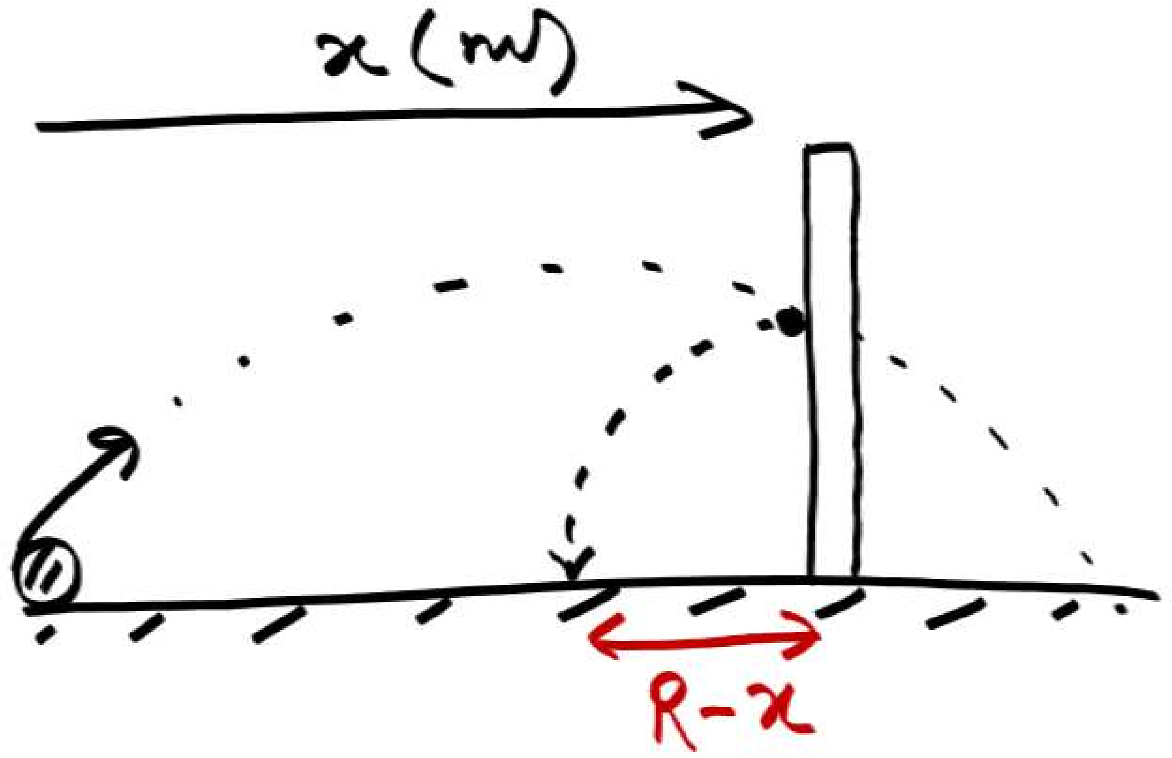
$$\frac{1}{\sqrt{3}} = \frac{x_1}{x_2}$$

Elastic collision of a projectile with a wall :

Case I : If $x \geq \frac{R}{2}$

Here distance of landing place of projectile from its point of projection is $2x - R$.



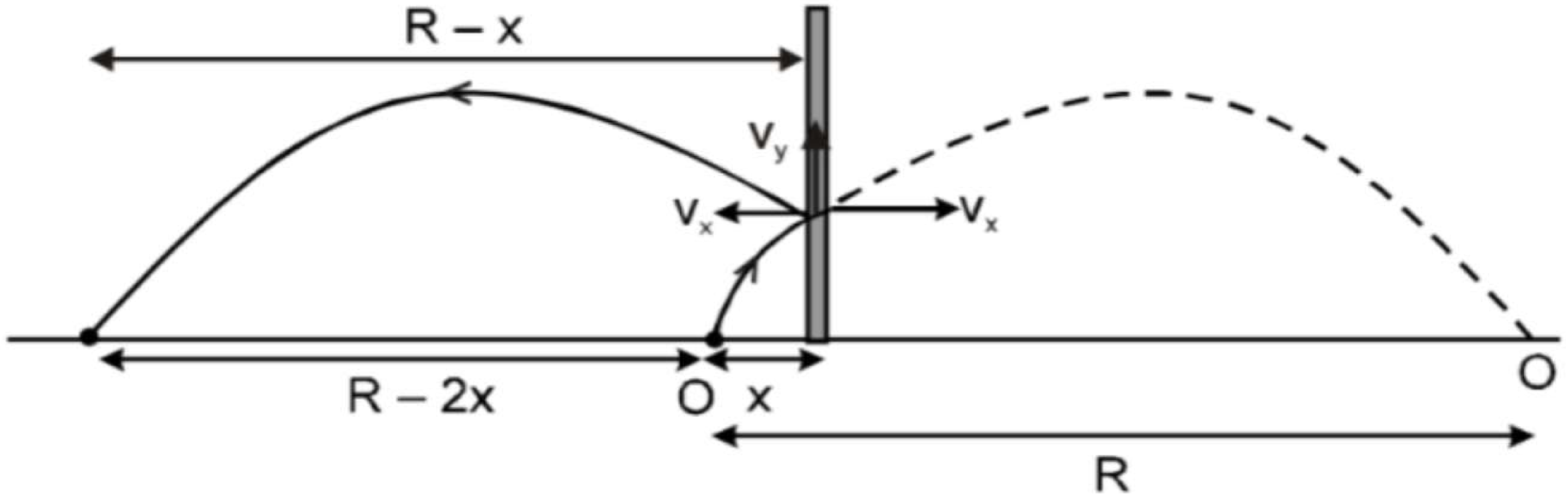


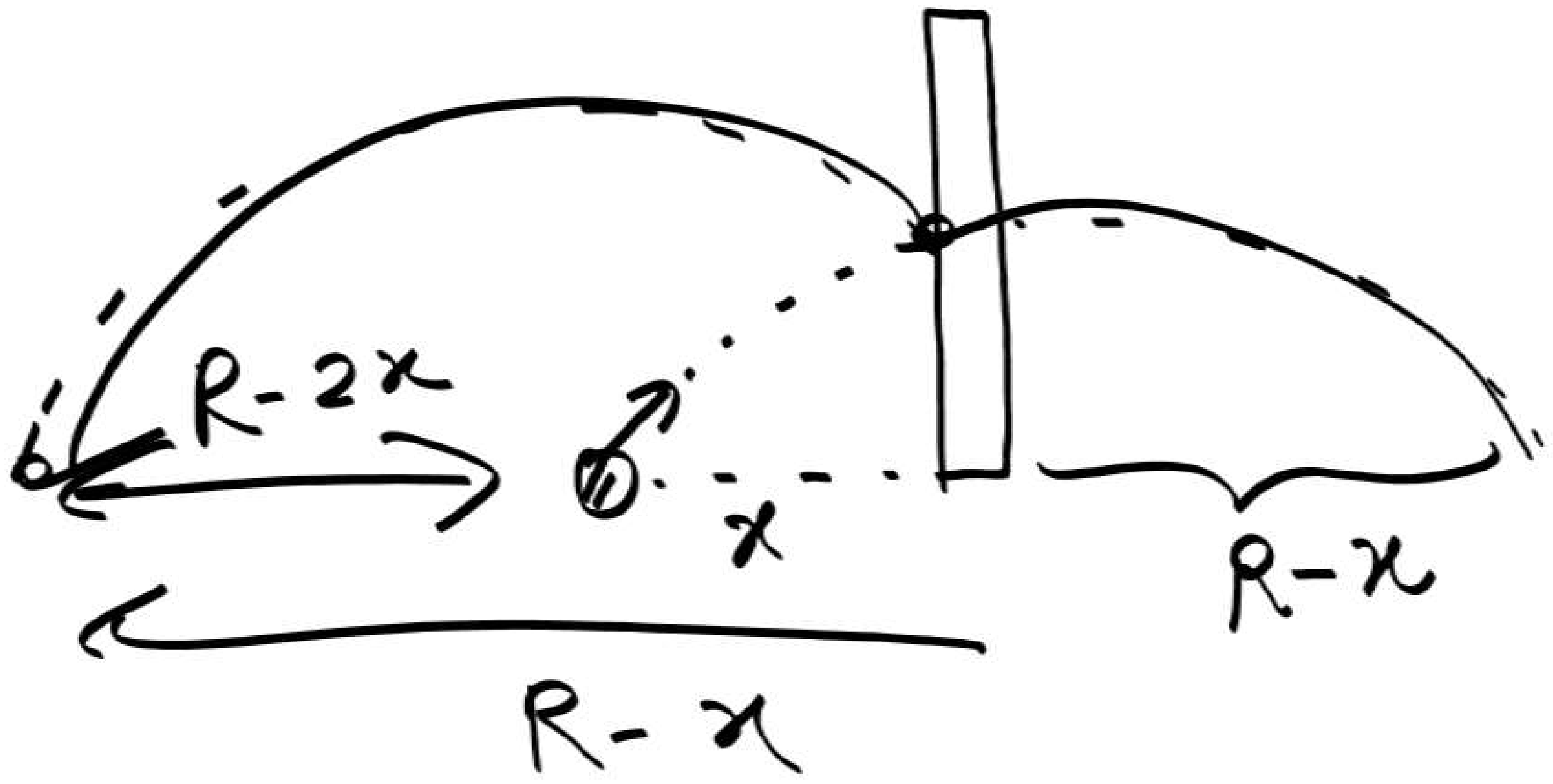
$x-(R-x)$

$2x-R$

Case II : If $x < \frac{R}{2}$

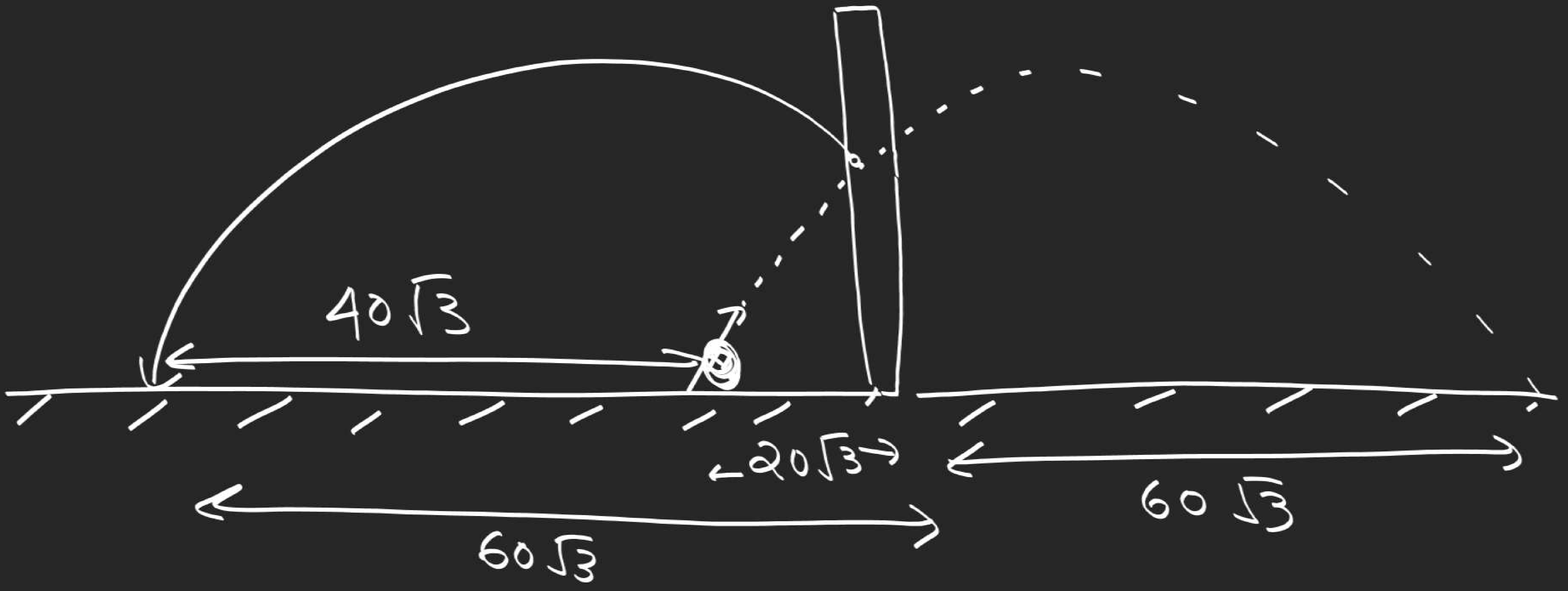
Here distance of landing place of projectile from its point of projection is $R - 2x$.





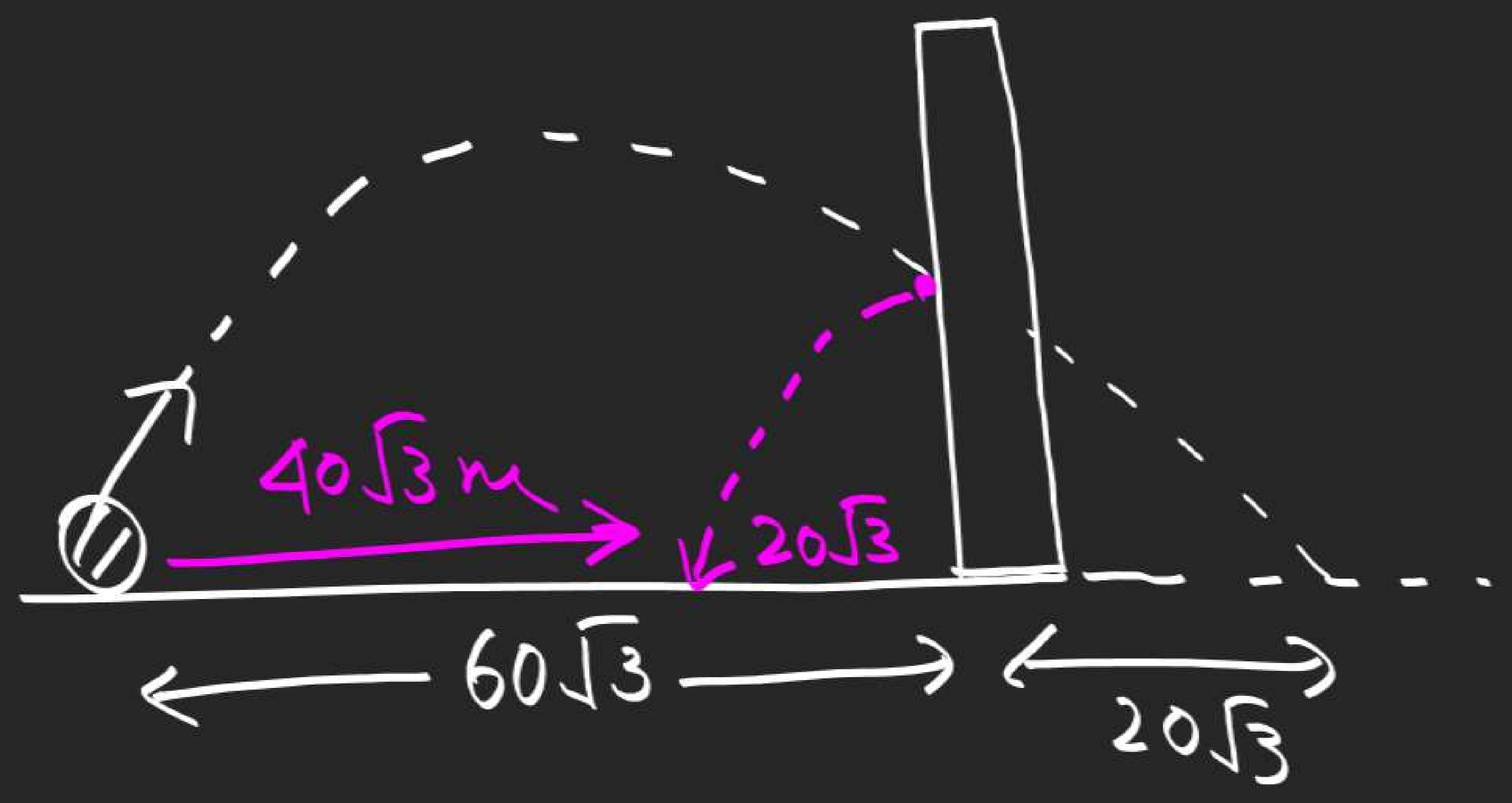
Q A ball thrown with velocity 40 m/s at 30° collides with wall at $20\sqrt{3}$ m. find the final posn of landing from point of Projection.

Ans Range $= 80\sqrt{3}$



Q A ball thrown with velocity 40 m/s at 30° collides with wall at $60\sqrt{3}$ m. find the final posⁿ of landing from point of Projection.

Ans $R = \frac{u^2 \sin 2\theta}{g} = \frac{1600}{10} \times \frac{\sqrt{3}}{2} = 80\sqrt{3}$



Relative Motion :

(i) Velocity of (A) wrt (B) $\Rightarrow$ $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$

ex $\vec{V}_A = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{V}_B = \hat{i} + \hat{j} - 2\hat{k}$

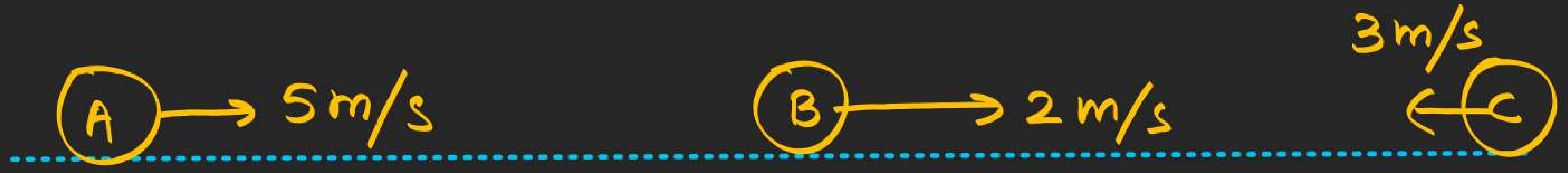
find magnitude of velocity of (A) wrt (B)

Ans $\vec{V}_{A/B} = (2\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} + \hat{j} - 2\hat{k})$

$\vec{V}_{A/B} = \hat{i} - 4\hat{j} + 3\hat{k}$

$|\vec{V}_{A/B}| = \sqrt{1 + 16 + 9}$

$= \sqrt{26}$ m/s



(i) $\vec{V}_{A|B} = \vec{V}_A - \vec{V}_B$

$= 5\hat{i} - 2\hat{i}$

$= 3\hat{i}$

$= 3\,m/s$

(ii) $\vec{V}_{B/C} = \vec{V}_B - \vec{V}_C$

$= 2\hat{i} - (-3\hat{i})$

$= 5\hat{i}$

$= 5\,m/s$

(A) moving 5 m/s towards east

(B) moving 5 m/s towards north.

then $\vec{V}_{A/B}$ is

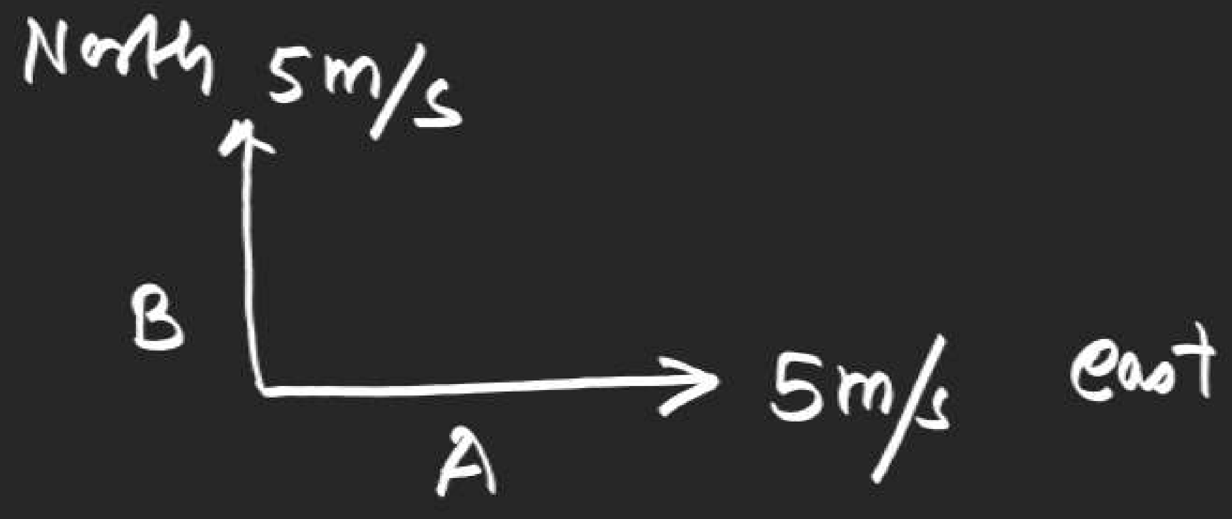
a) $5\sqrt{2}$ m/s towards North - east

b) $5\sqrt{2}$ m/s ——— west - north

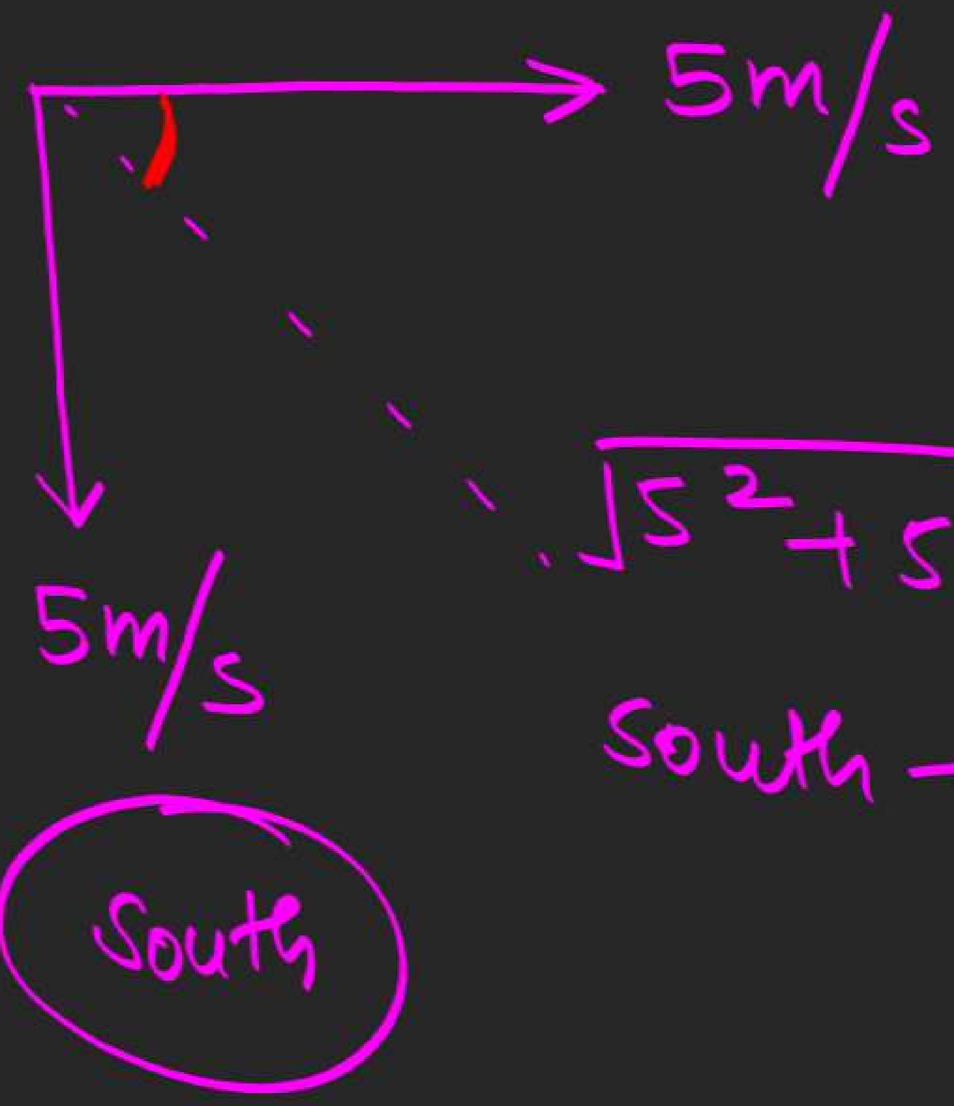
~~c)~~ $5\sqrt{2}$ m/s ——— south - east

d) None

Ans : Given :



$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$



$\sqrt{5^2+5^2} = 5\sqrt{2}$

south - east

Q

In how much time (A) will catch (B)

Ans

A $V_r = 10 - 20 = -10\,m/s$

$a_r = 4 - 1 = 3\,m/s^2$

$S = 30$

$S = ut + \frac{1}{2}at^2$

$30 = -10t + \frac{1}{2}(3)t^2$

$3t^2 - 20t - 60 = 0$

$t = \frac{20 \pm \sqrt{1120}}{6} = \frac{20 + 33.5}{6} \approx 9\,sec.$

Eqn of motion

Identify main object

then calculate relative

$\boxed{V_r,\ a_r,\ S_r}$

Keeping other stationary

Giving its V, a to main object with

$\ominus$ sign

Ans

A $\rightarrow$ $v_r = 10 - (-10) = 20\,m/s$

$a_r = 2 - (-3) = 5\,m/s^2$

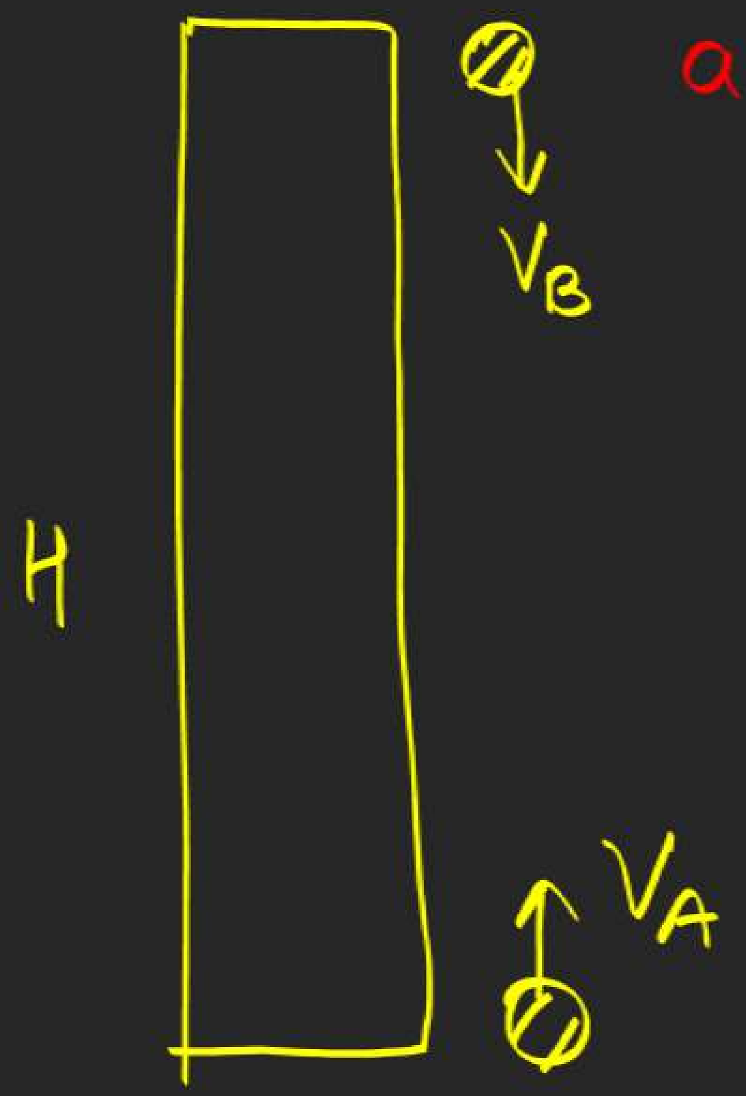
$$s = ut + \frac{1}{2}at^2$$

$$100 = 20t + \frac{1}{2}(5)t^2$$

$$5t^2 + 40t - 200 = 0$$

$$t^2 + 8t - 40 = 0 \longrightarrow t = ?$$

Motion Under Gravity



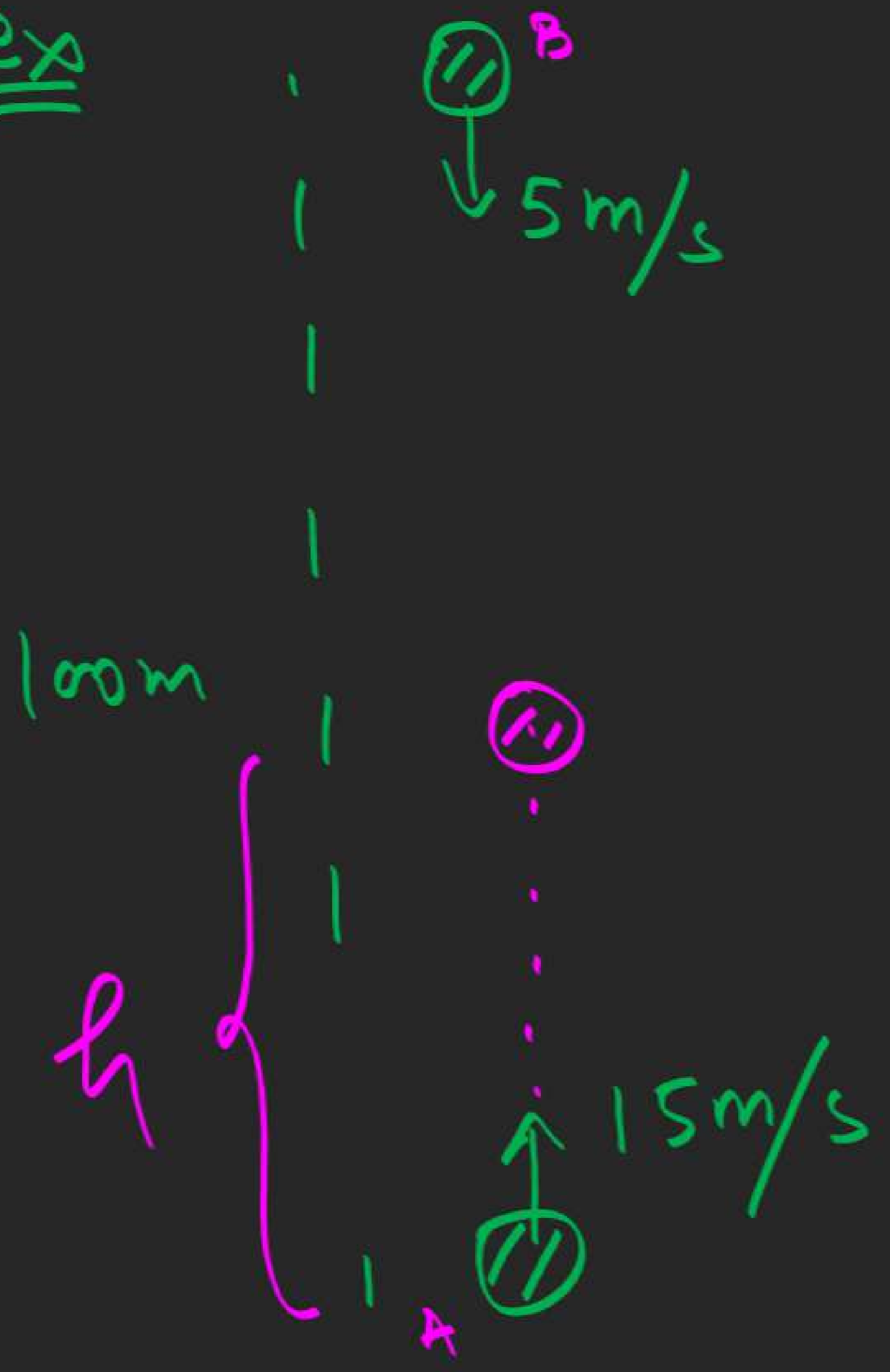
$a_B = -g$

$a_A = -g$

$\Rightarrow a_r = 0$

$\Rightarrow \boxed{H = V_r t}$

ex



i) V_r of A $= 20\,m/s$

$H = V_r \cdot t$

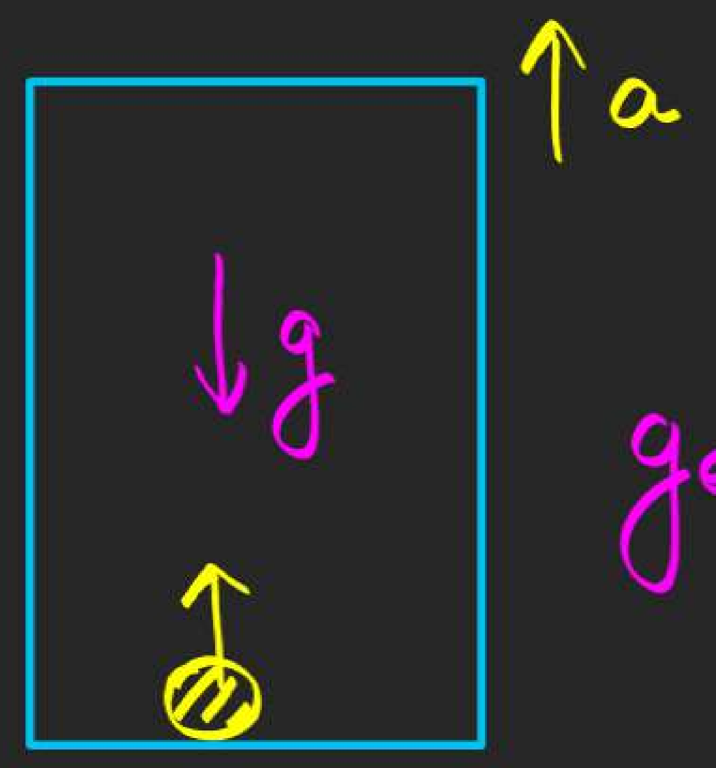
$100 = 20\,t$

$\boxed{5 = \text{time}}$

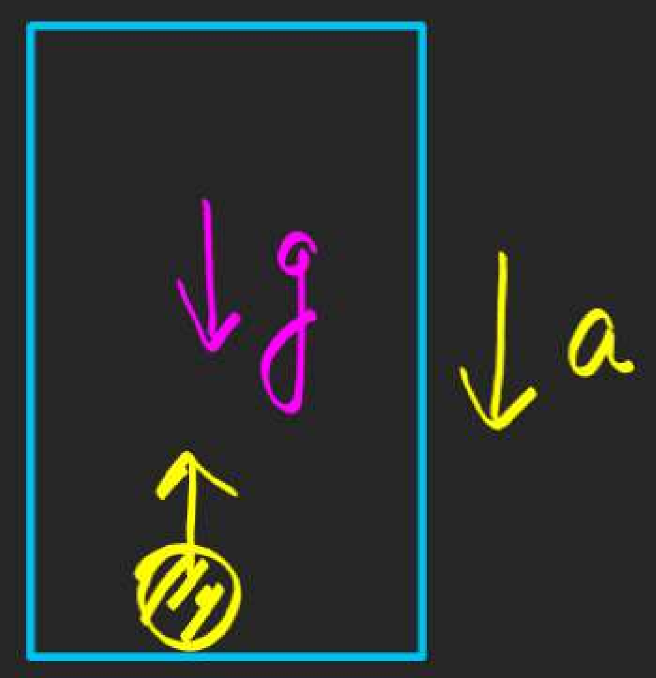
ii) where it collided

$h = U_A t - \frac{1}{2} g t^2$

In the lift



$g_{eff} = (g+a)$

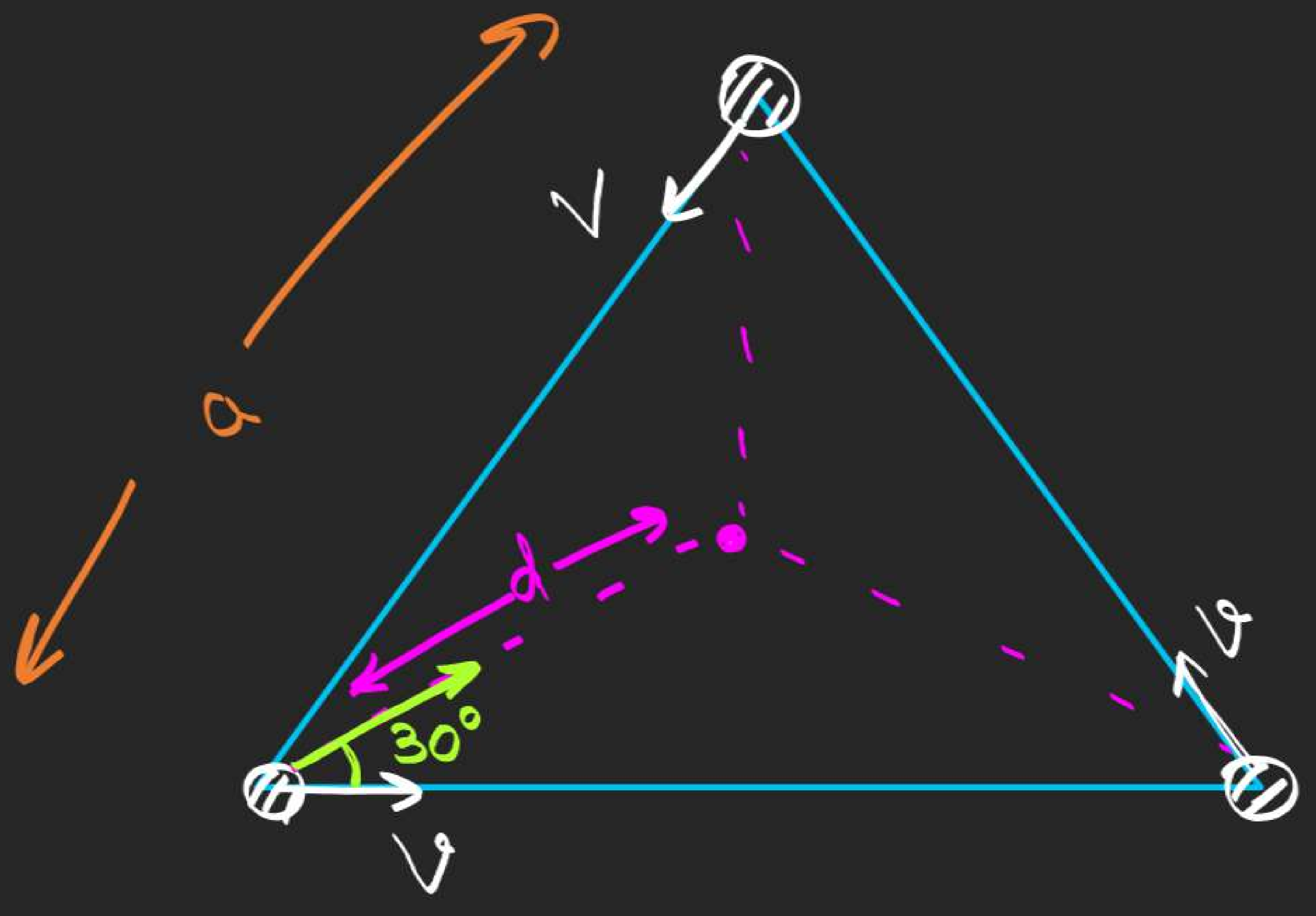


$g_{eff} = (g-a)$

everything is same ——"——

$\Rightarrow$ use $\Rightarrow$ g_{eff} in place of g

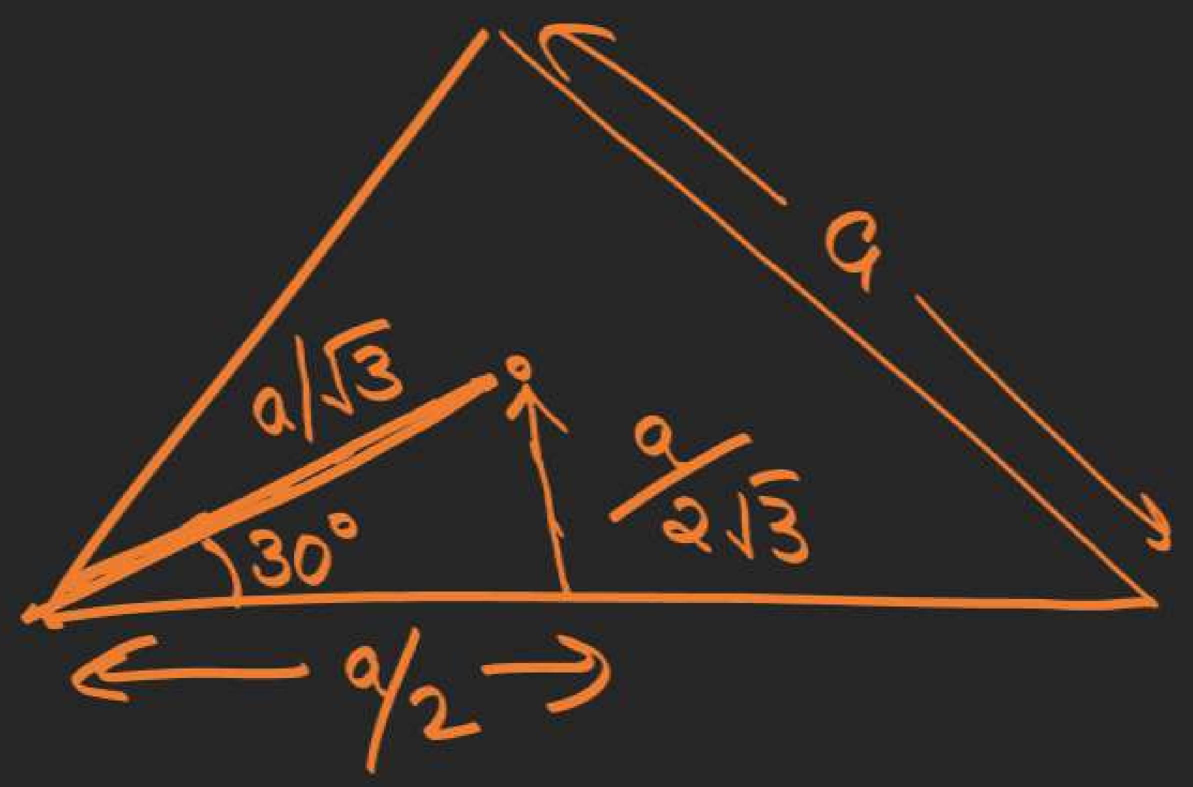
when three object collide/meet on centre of Δ

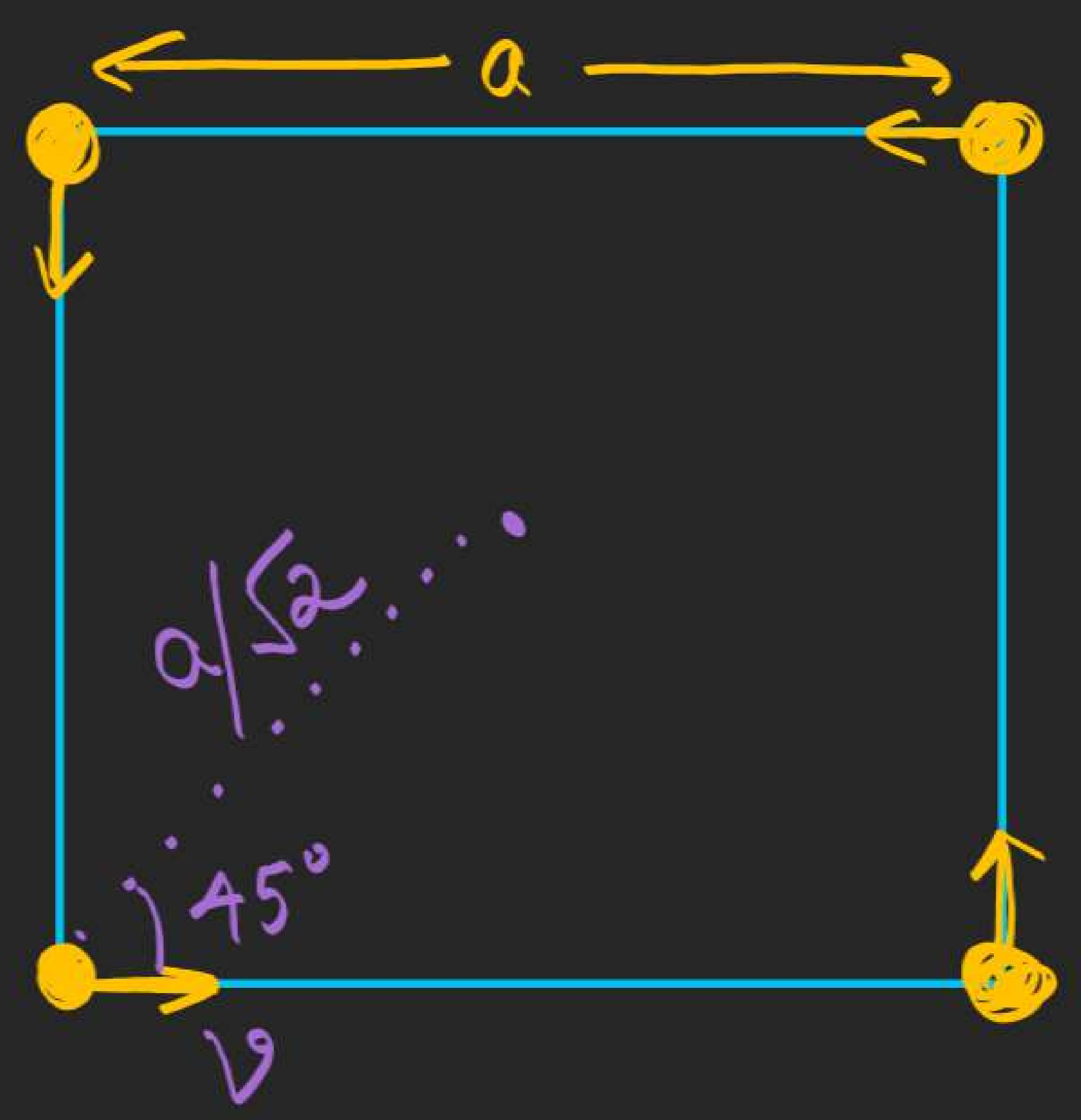


$$\Rightarrow \frac{d}{V_{\text{along the Point}}} = \frac{a/\sqrt{3}}{V\cos 30^\circ}$$

$$= \frac{a}{\sqrt{3} \times V\frac{\sqrt{3}}{2}}$$

$$t = \frac{2a}{3V}$$

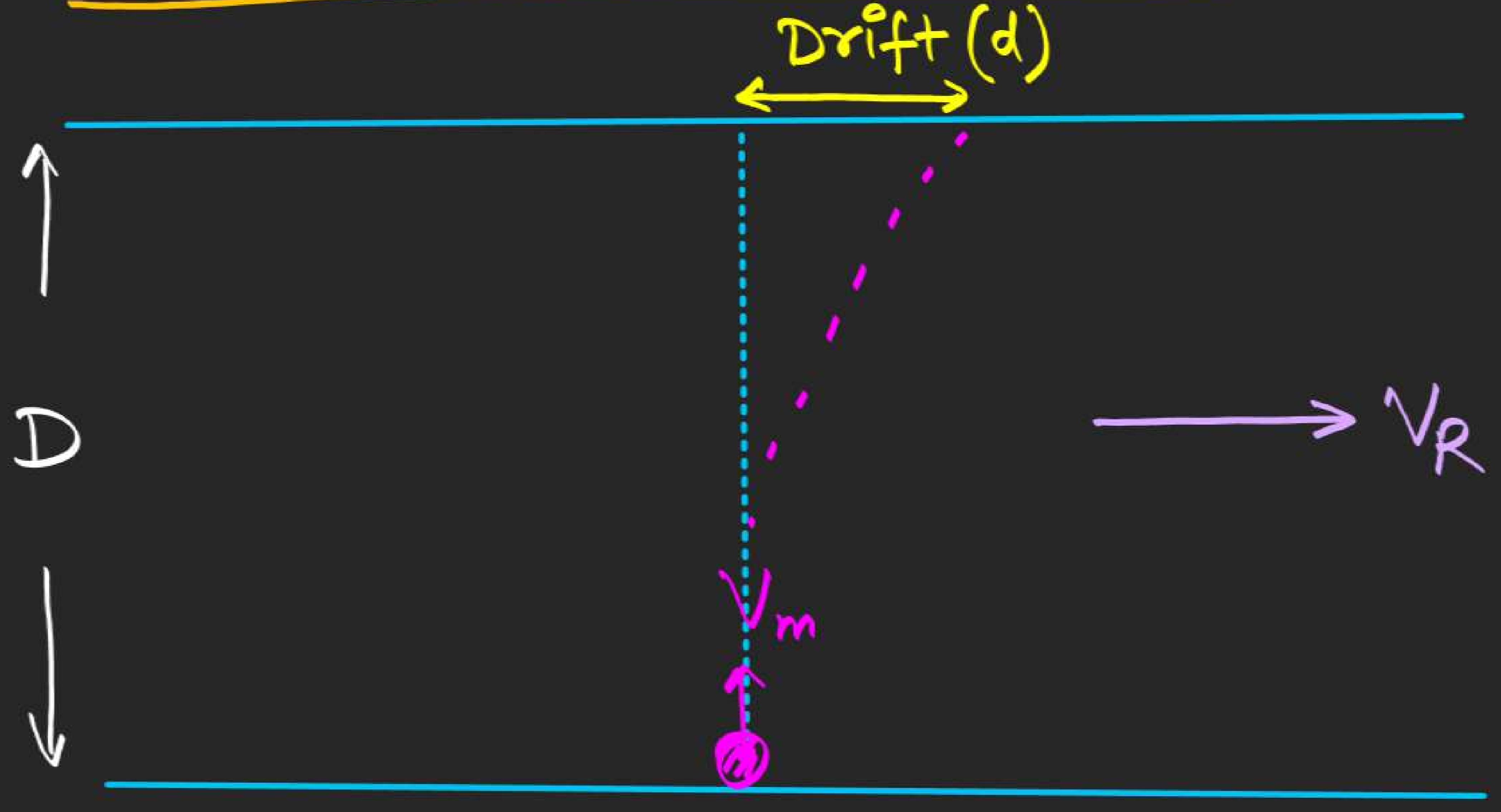




$$t = \frac{d}{V_{\text{along centre}}} = \frac{a/\sqrt{2}}{V\cos 45^\circ} = \frac{a}{v}$$

$$d = \frac{\sqrt{2}a}{2}$$
$$= a/\sqrt{2}$$

River - Swimmer Problems

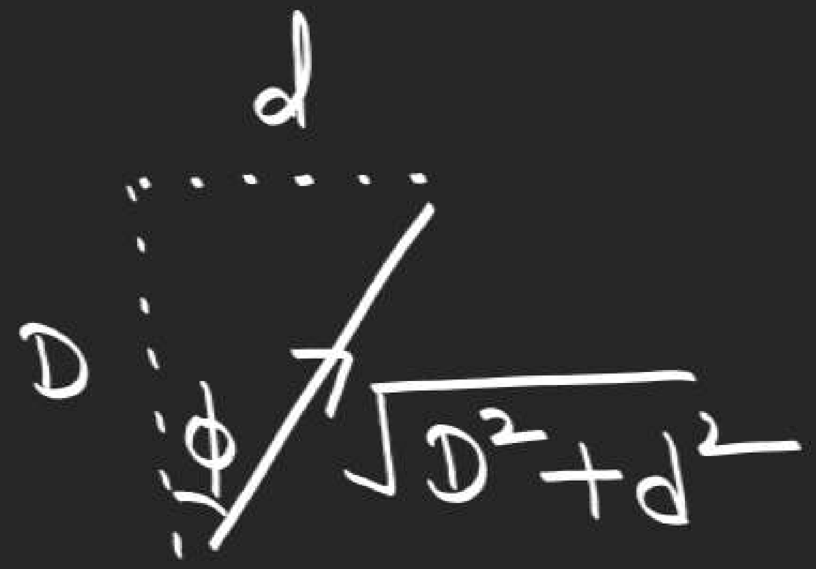


Case of min time : $\boxed{t_{min} = \dfrac{D}{V_m}}$

→ suffer drift (d)

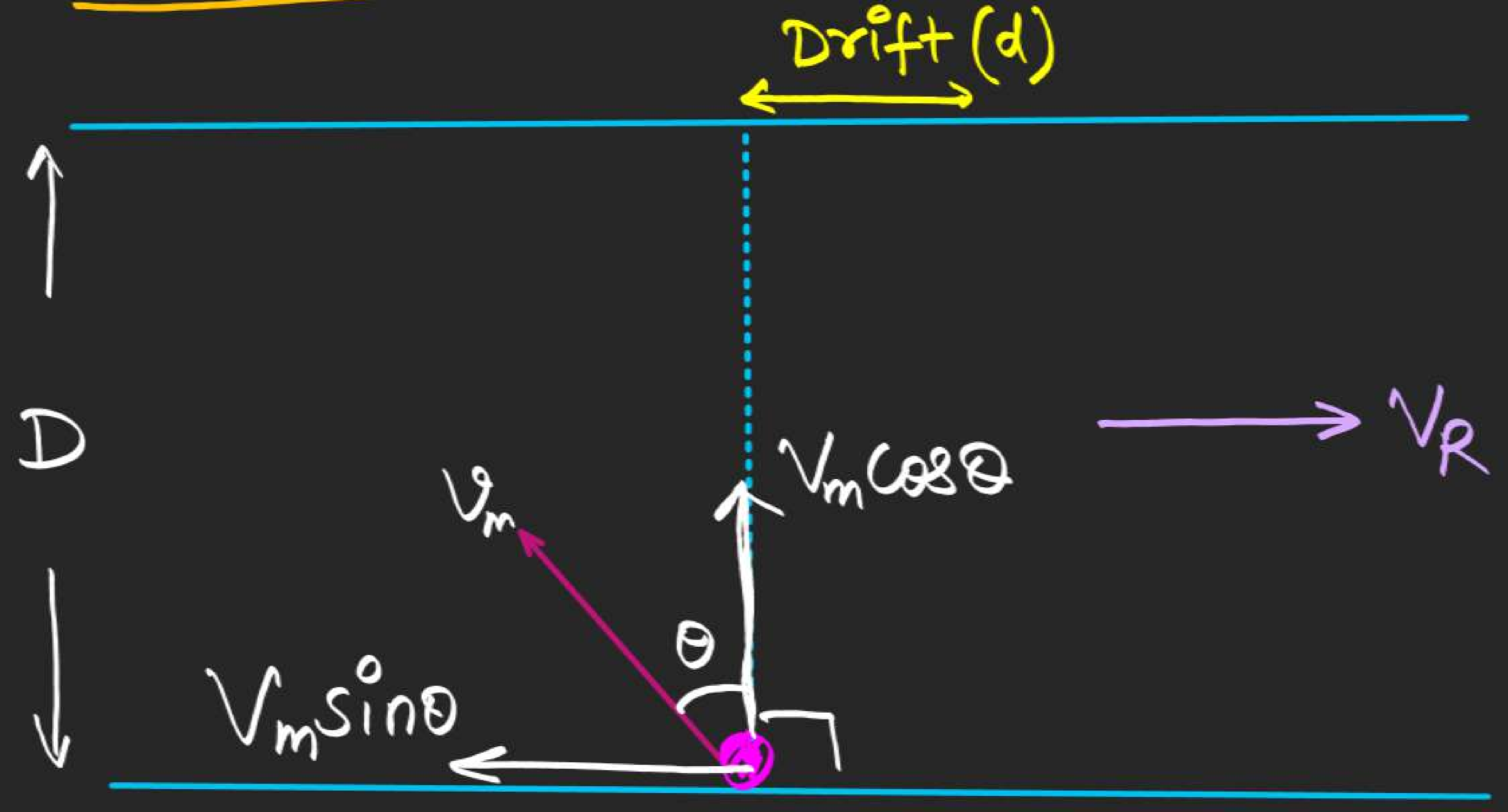
$$\boxed{d = V_R \cdot t}$$

→ displacement



$$\tan\phi = \left(\frac{d}{D}\right)$$

River - Swimmer Problems



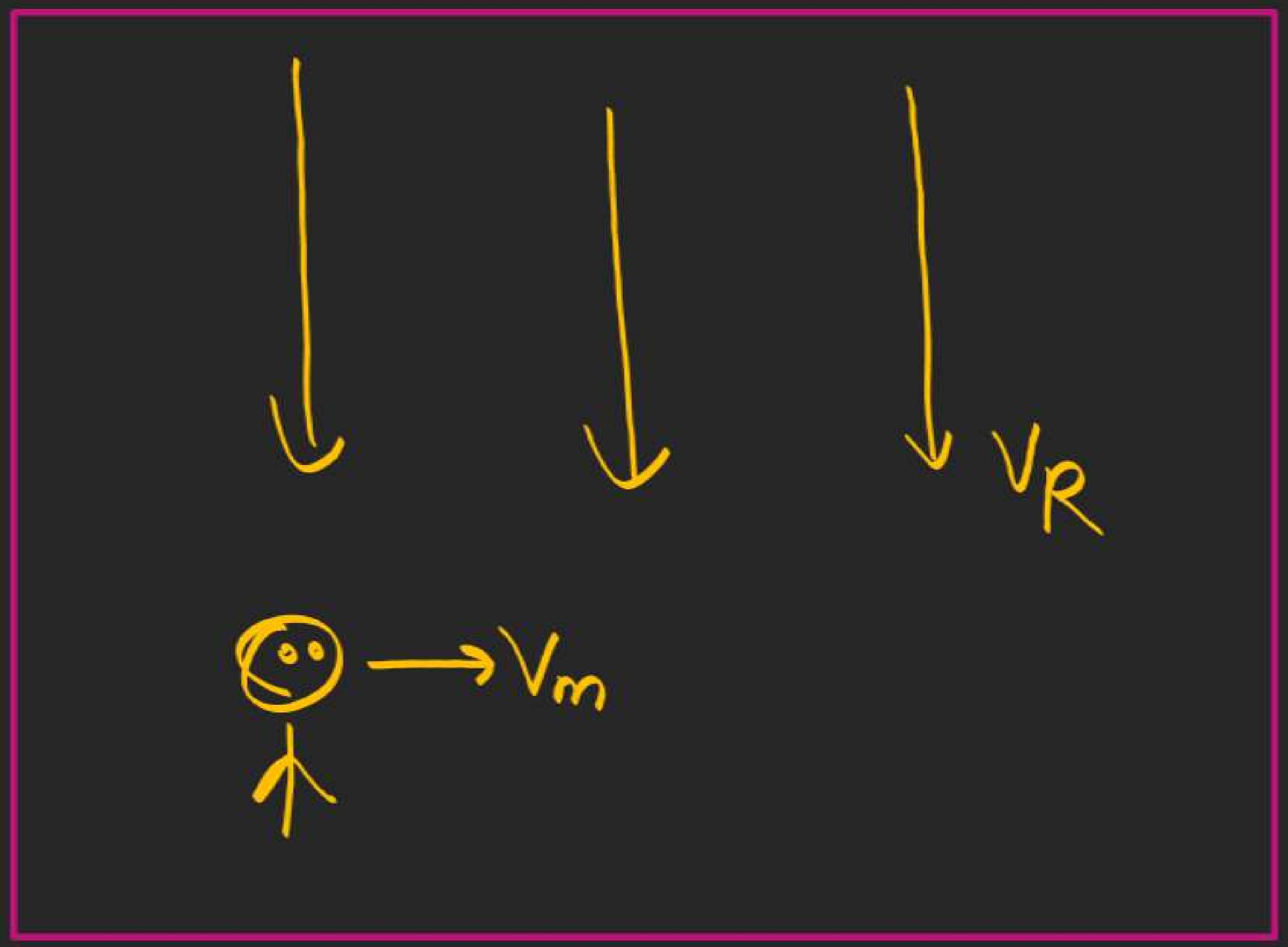
Case of zero drift / shortest Path

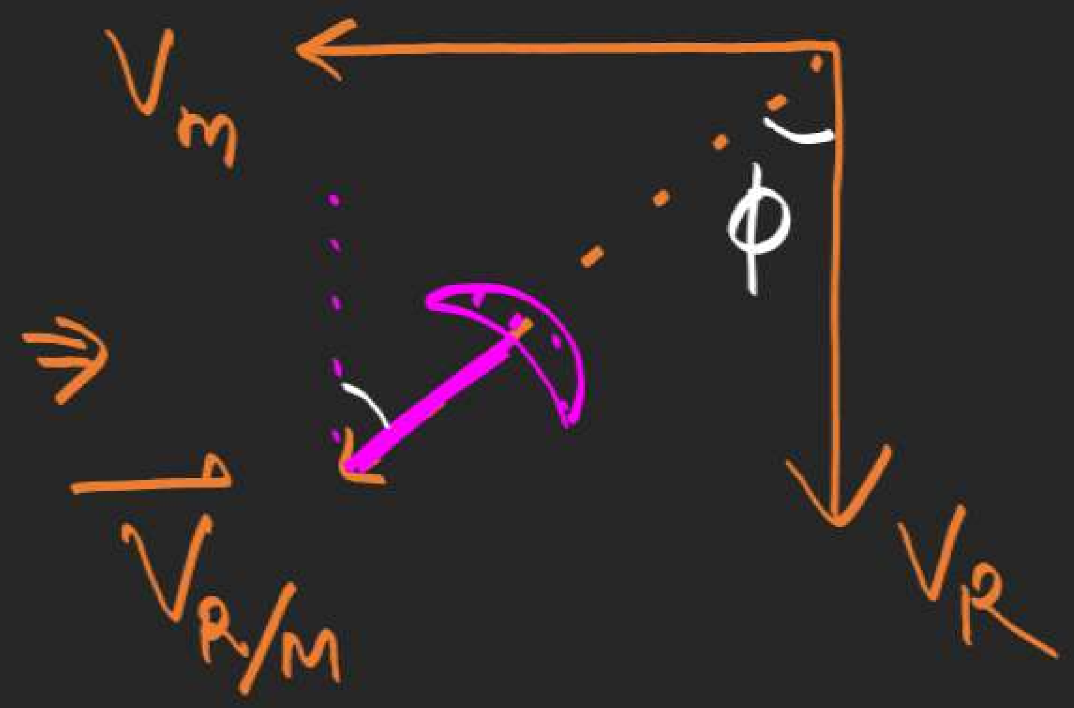
(i) From fig: $V_m \sin\theta = V_R$

(ii) $t = \dfrac{D}{V_m \cos\theta}$

Rain Umbrella Problem

ex

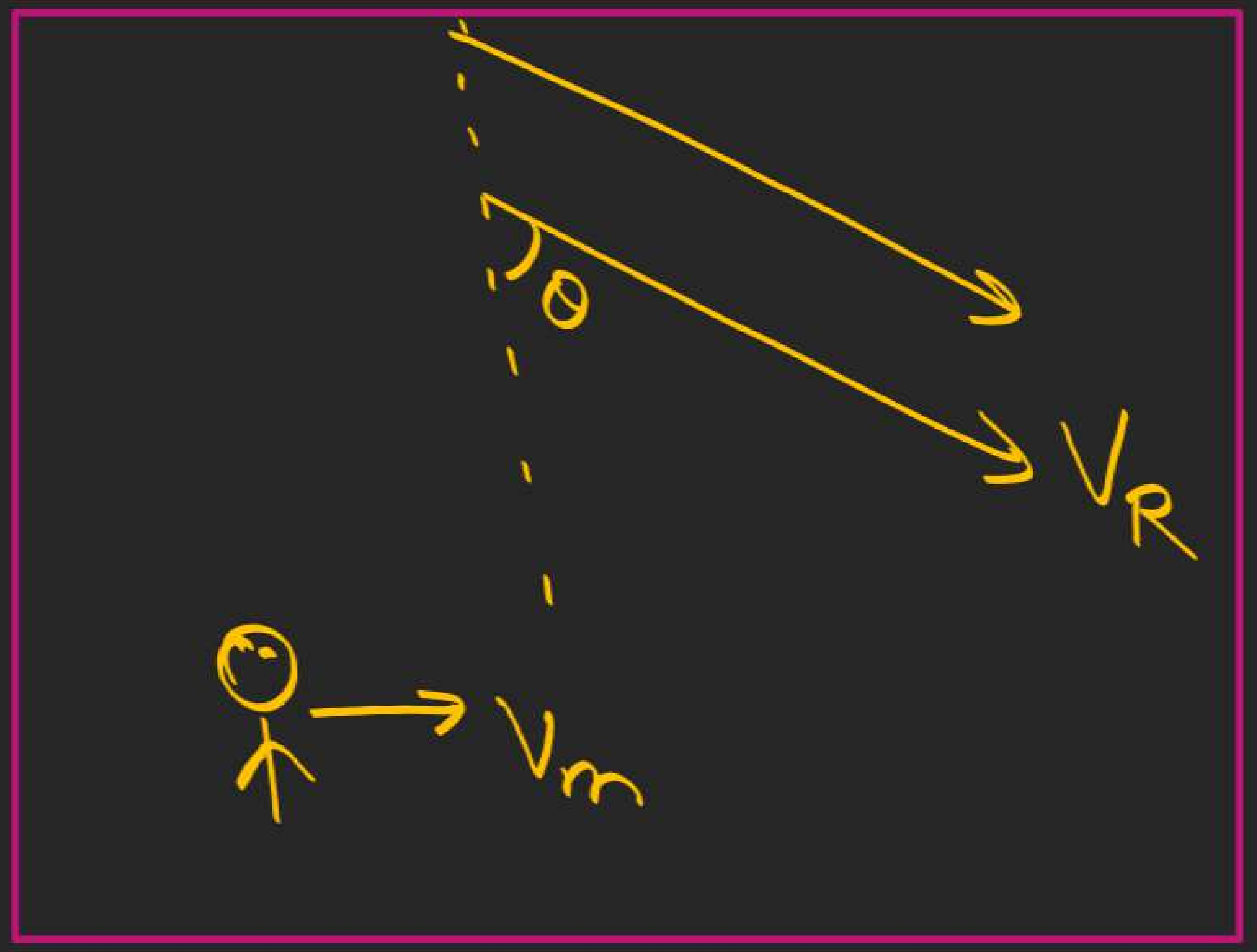


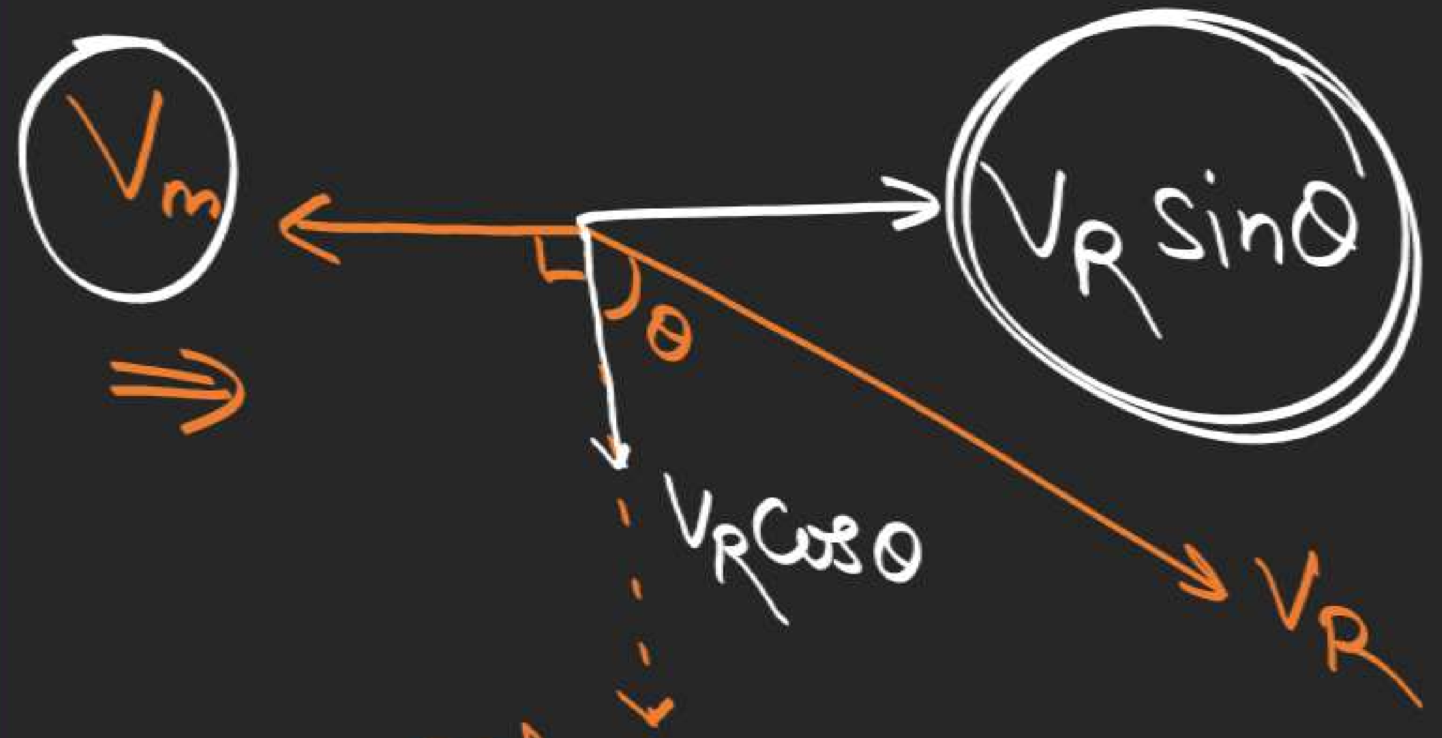


$$\tan\phi = \frac{V_m}{V_R}$$

$$\vec{V}_{R/M} = \sqrt{V_R^2 + V_m^2}$$

ex





$$\vec{V}_{R/M} = V_R \cos\theta$$

Find angle with vertical which umbrella should be held if ,

$V_r = 6\,\hat{i} - 8\,\hat{j}$.   Case:2 $V_G = 6\hat{i}$

$$\vec{V}_{R/M} = \vec{V}_R - \vec{V}_m$$

$$= (6\hat{i} - 8\hat{j}) - (6\hat{i})$$

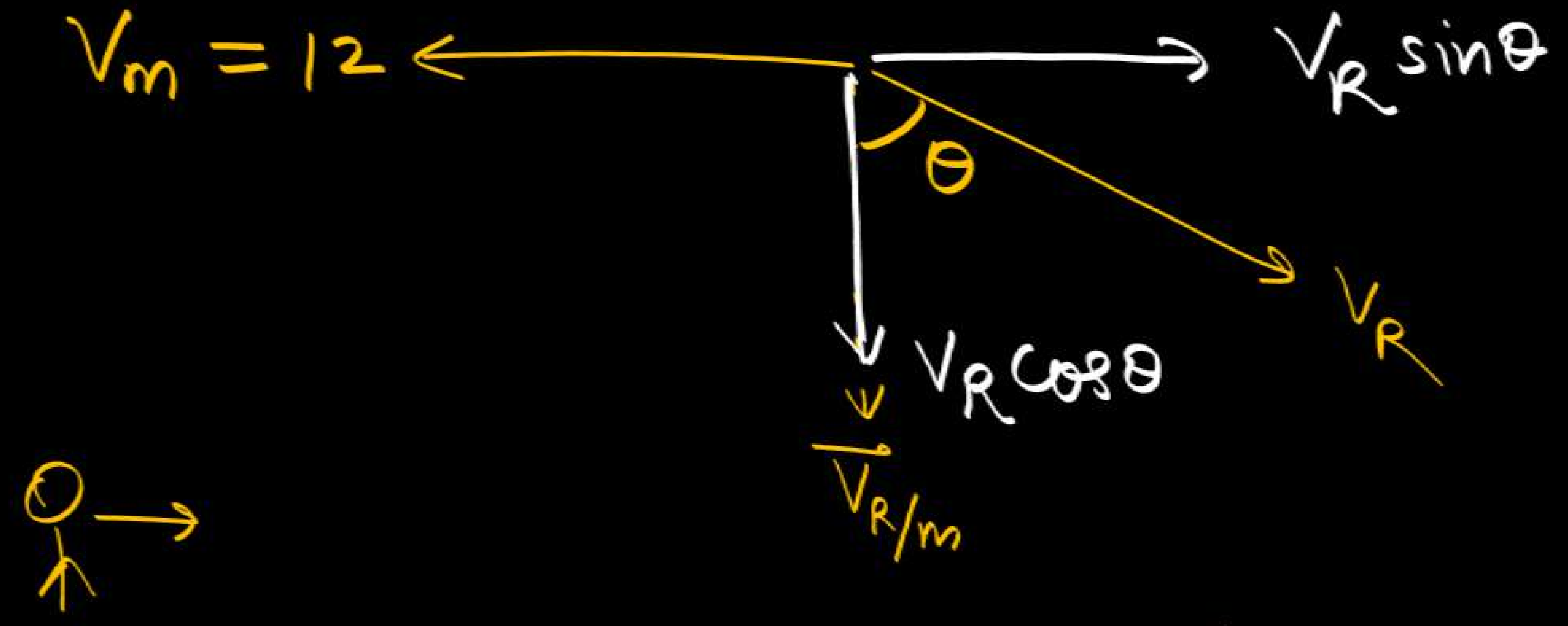
$$= -8\hat{j}$$

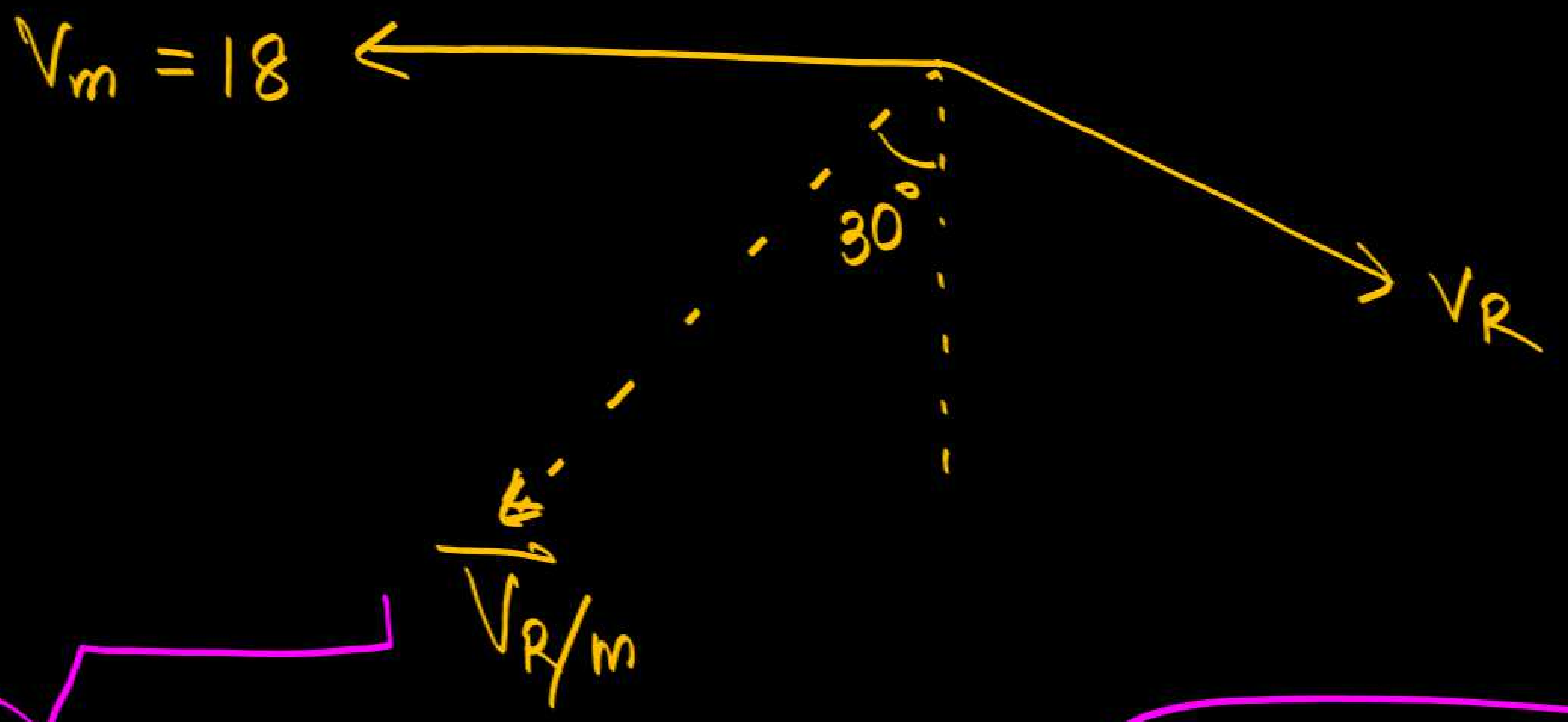
$$= 8\ m/s$$

$-8\ m/s$

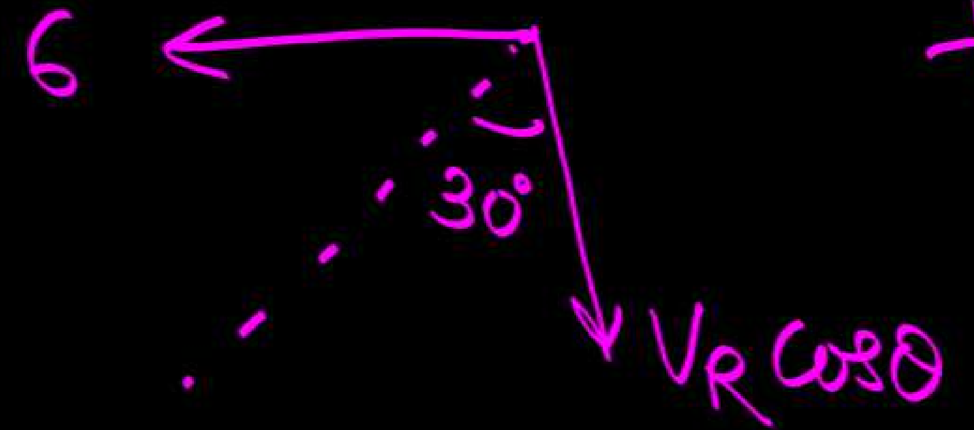
LIVE

A man running at 12 km/h on a horizontal road finds the rain falling vertically. He increases his speed to 18 km/h and finds that the drops make angle 30° with the vertical. Find the speed and direction of the rain w.r.t the road.





$$\boxed{V_R \sin\theta = 12}$$



$$\tan 30° = \frac{6}{V_R \cos\theta} \Rightarrow \boxed{V_R \cos\theta = 6\sqrt{3}}$$

square & add

$$(V_R)^2\left[\sin^2\theta + \cos^2\theta\right] = 144 + 108$$

$$V_R^2 = 252 \simeq 16 \text{ Km/h} = V_R$$